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THE PESTALOZZIAN SERIES OF SCHOOL ARITHMETICS.—PART II.

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THE

# DECIMAL SYSTEM OF NUMBERS;

ILLUSTRATED

AND

PRACTICALLY APPLIED

IN A SERIES OF

SYSTEMATIC AND PROGRESSIVE EXERCISES.

BY DANA P. COLBURN,

PRINCIPAL OF THE MATHEMATICAL DEPARTMENT OF THE NEW ENGLAND NORMAL INSTITUTE, LANCASTER, MASS.; AUTHOR OF "INTEREST, DISCOUNT AND EQUATION OF PAYMENTS;" AND CO-AUTHOR OF "FIRST STEPS IN NUMBERS."

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# PREFACE.

This work has been prepared to follow the "First Steps in Numbers" by D. P. Colburn and G. A. Walton, and is designed to give the student in it so complete a mastery over the elementary operations on numbers, as shall enable him to add, subtract, multiply, or divide, any numbers whatever, with ease, accuracy and rapidity. It is also designed to give him such thorough mental training as shall enable him to solve any arithmetical problem that may arise in practical life, and fit him to pursue advantageously the higher departments of arithmetic, or the other mathematical studies.

It was originally commenced in connection with Mr. Geo. A. Walton, my associate in a former work, but, by a subsequent arrangement, I assumed the authorship and entire direction of it, retaining however the privilege of using the materials that had been contributed by him. Many of the practical examples were furnished by Mr. Walton, to whom I am also indebted for many suggestions, and for assistance in revising and preparing the manuscript for the press.

To no one am I so much indebted for any valuable arithmetical ideas which I may possess, as to my former teacher, N. Tillinghast, Esq., Principal of the Bridgewater State Normal School. Those who best know him, will best understand how great that indebtedness must be.

The plan and arrangement of this treatise are, however, entirely original with me, and for its defects, I alone must be held responsible.

Dana P. Colburn.

# INTRODUCTION.

## EXHIBITING NATURE OF THE DECIMAL SYSTEM OF REPRESENTING NUMBERS, AND PLAN OF THE PESTALOZZIAN SERIES OF ARITHMETICS.

When looking at a small group of persons (as one of three, four or five), we can easily distinguish all the individuals composing it; but if the group should gradually increase, it would become more and more difficult to distinguish its individual members; and should it become very large, this difficulty would be almost insurmountable. To be convinced of this, we have only to compare the ideas we form of a group of three persons with those we form of a group of one hundred. We shall find that while we can easily distinguish all the members of the first group, we can only see the second as a whole, without being able to form distinct mental pictures of all its individual members.

Now, were there no method of conceiving of numbers except by direct reference to the simple units which they include, large numbers could never be used. The mind would endeavor in vain to grasp them, and even were the conception possible, no adequate method of representing them could be devised. But the necessity or desire of using them would naturally lead to some invention for overcoming these difficulties, as it probably did lead to the invention of our present Decimal System.

The fundamental idea of that system, the idea on which its simplicity, perfectness, and beauty depend, is that a certain number of individuals are regarded as forming a single collection or group ; one more than this number, as a group and one individual ; two more, as a group and two ; three more, as a group and three ; and so on, till we have two groups, two groups and one, two groups and two, &c. &c.

Thus, if eight individuals were taken as a convenient number for a group, nine would be regarded as *eight and one ;* ten as *eight and two ;* fifteen as *eight and seven ;* sixteen as *two eights ;* seventeen as *two eights and one ;* and so on, till we reach *eight eights.* But precisely the same principle which would lead us to regard eight individuals as a single group, would lead us to regard these eight groups as forming a larger group of sixty-four ; and eight groups of sixty-four each, as a new group of five hundred and twelve ; and so on, every eight groups of one size would give a new group of a larger size.

Large numbers being thus conceived of as made up of groups, such names would be invented for them as would best exhibit our idea of their nature. If eight were the base, distinct primitive names would be given to the first eight numbers, and to the number contained in each group, but derived names would be given to all the others. Adopting, for convenience of illustration, our present familiar names as far as possible, we might count thus: *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*; *eight and one*, *eight and two*, *eight and three*, *&c.*, *to eight and seven*, *two eights*, *two eights and one*, *two eights and two*, *&c.*, as far as *eight eights*; to which, as it forms a new group, we should give a new name. Calling it *sixty-four*, we should count on thus: *sixty-four and one*, *sixty-four and two*, *sixty-four and three*, &c., to *sixty-four and eight*; *sixty-four*, *eight*, *and one*; *sixty-four*, *eight*, *and two*, &c., to *sixty-four*, *seven eights*, *and seven*; *two sixty-fours*; *two sixty-fours and one*, &c., to *seven sixty-fours*, *seven eights*, *and seven*; *eight sixty-fours*, or a new group of five hundred and twelve; from which we should count on the same principle as before.

If this method were generally adopted, we might naturally expect that abbreviated expressions would soon be used instead of the forms given above. By dropping the conjunctions and forming compound words, we might count from eight thus: *eight-one*, *eight-two*, *eight-three*, &c., to *two-eights*, *two-eights-one*, *two-eights-two*, *two-eights-three*, &c., to *seven-eights-seven*, *sixty-four*, *sixty-four and one*, *sixty-four and two*, &c., to *sixty-four and eight*, *sixty-four and eight-one*, *sixty-four and eight-two*, &c., to *two sixty-fours and one*, *two sixty-fours and two*, &c., to *seven sixty-fours and seven*, *five hundred and twelve*, *five hundred and twelve and one*, &c. &c.

If characters to represent numbers were now to be invented, they would probably correspond to the system we have thus far developed. A peculiar character would be required for each number till we reach that which is the base of the system; after which no new characters (save those to mark place) would be needed, as each higher number could be represented by so arranging and combining the former characters as to indicate the groups composing it. For instance, the denomination or group represented by any figure might be determined by its position with reference to some fixed starting-point. Suppose that a dot or period should be used for this purpose, and that it should be agreed upon that the figure immediately at the left of the dot should represent units; that the second figure at the left should represent eights; the third, sixty-fours; the fourth, five hundred and twelves, &c. We could then, by adopting seven figures, as, for instance, the first seven of our Decimal System, represent numbers as below:

57.=five-eights-seven.
63.=six-eights-three.
432.=four sixty-fours, and three-eights-two.
666.=six sixty-fours, and six-eights-six.

If we should not wish to represent units and all the intervening groups to the highest considered, a character would be needed to mark the spaces left vacant; for otherwise, it might be difficult to tell what denomination some of the figures are intended to represent. For instance, if *four sixty-fours* were to be written thus, 4 ., we might be in doubt whether to call it four eights or four sixty-fours; but, if the two places between the four and the point were to be occupied by same character, as a star, the difficulty would be obviated. We should then have,

5*.=five-eights.
2**.=two sixty-fours.
3*7=three sixty-fours and seven,
&c. &c.

It is obvious that, in this system, the denomination represented by a figure in any place is eight times what it would be were the figure written one place further to the right, and one-eighth of what it would be if written one place further to the left — sixty-four times what it would be if written two places further to the right, and one sixty-fourth of what it would be if written two places further to the left. We could then multiply and divide by eight and the powers of eight, as we now multiply and divide by ten and the powers of ten, and could represent *octonary* fractions in precisely the same way as we usually represent *decimal* ones. For instance, we should have:

4.2=four and two-eighths
.*5=five sixty-fourths.
3*.*7=three-eights, and seven sixty-fourths.

If each group had consisted of *nine* instead of *eight*, we should have had primitive names for the numbers as far as nine, and derivatives for the others, thus: *nine-one*, *nine-two*, *nine-three*, &c., *two-nines*, *two-nines-one*, *two-nines-two*, &c., *eight-nines-eight*, *eighty-one*, *eighty-one and one*, *eighty-one and two; two eighty-ones, and four-nines-seven; two eighty-ones*, *and four-nines-eight*, &c. &c. In writing them by figures, we should need, besides the dot to mark the starting-point, and the character to fill spaces which would otherwise be left vacant, a character for each number below nine. Then would

1*.=one-nine, or nine.
43.=four-nines-three.
578.=five-eighty-ones and seven-nines-eight.
326.=three eighty-ones and two-nines-six.
4**.=four eighty-ones.
8**.=eight eighty-ones.
4.7=four and seven-ninths.
3.*6=three and six eighty-firsts.

If each group had consisted of *ten*, we should, on the same principle, have had derivative names for the numbers above ten, thus: *ten-one*, *ten-two*, *ten-three*, &c., to *ten-nine*, *two-tens*, *two-tens-one*, *two-tens-two*, &c., *nine-tens-nine*, *one hundred*, *one hundred and one*, &c., *one hundred and four-tens-six*, *one hundred and four-tens-seven*, &c. &c.

If, in forming the derivative names of the numbers from ten to twenty, we had changed the order of the primitive words, we should have counted from ten as follows: *one-ten*, *two-ten*, *three-ten*, *four-ten*, *five-ten*, *six-ten*, *seven-ten*, *eight-ten*, *nine-ten*, *two-tens*, *two-tens-one*, &c. &c.

The system, thus modified, coming into general use, further changes and corruptions would be likely to be made in the pronunciation of the names it employs. Changing the *ten* to *teen*, and omitting the hyphen, would give *oneteen*, *twoteen*, *threeteen*, *fourteen*, *fiveteen*, *sixteen*, *seventeen*, *eighteen*, *and nineteen*. Again, changing the *five* to *fif* and the *three* to *thir*, for the sake of euphony, and then substituting for *one-teen* and *two-teen* *eleven* and *twelve*, derived from Saxon words signifying respectively *one left* and *two left*, we should have the common names of the numbers from ten to twenty.

If we substitute *ty* for *ten* in the words *two-tens*, *three-tens* &c., we shall have *twoty*, *threety*, *fourty*, *fivety*, *sixty*, *seventy*, *eighty*, *ninety*, If we should now change the *three* to *thir*, the *four* to *for*, and the *five* to *fif*, and substitute *twen*, derived from *twain*, for the *two*, we should have the familiar names *twenty*, *thirty*, *forty*, and *fifty*.

These principles extended, give the *Decimal* System of Numbers. In applying figures to this system, we should need characters for each number below ten, a character to fill vacant places, and a dot (decimal point) to mark the starting-point.

We should then have

1*.=ten.
5*.=fifty.
63.=sixty-three.
8**.=eight hundred.
8*7.=eight hundred and seven.

Or, using a zero to mark the place of the omitted denominations, we should have the common method of writing numbers.

Such is the Decimal System of representing Numbers, — probably the most simple and perfect thing of man's invention. It is a system so simple that a child can understand it, yet so comprehensive that it includes all possible numbers, and represents them with only ten simple characters and a point.

Moreover, it makes all numerical operations depend on the combinations of which the ten primitive numbers are susceptible, so that a person has only to learn these combinations, and the method of applying them to the system itself, to be a perfect master of all numerical work. To illustrate this, we insert the following table of all the combinations, in Addition, of which the primitive numbers are susceptible.

$2=1+1$
$3=2+1$
$4=3+1=2+2$
$5=4+1=3+2$
$6=5+1=4+2=3+3$
$7=6+1=5+2=4+3$
$8=7+1=6+2=5+3=4+4$
$9=8+1=7+2=6+3=5+4$
$10=9+1=8+2=7+3=6+4=5+5$
$11=10+1=9+2=8+3=7+4=6+5$
$12=10+2=9+3=8+4=7+5=6+6$
$13=10+3=9+4=8+5=7+6$
$14=10+4=9+5=8+6=7+7$
$15=10+5=9+6=8+7$
$16=10+6=9+7=8+8$
$17=10+7=9+8$
$18=10+8=9+9$
$19=10+9$
$20=10+10$

It is obvious that the only combination of integers which will give 2 for a sum is 1+1. The only combinations which will give 3 are 2+1, 1+2, and 1+1+1. But 2+1 and 1+2 are so essentially the same operation, that a person who understands one must, of necessity, understand the other. To add 1+1+1, we first add 1+1, which but repeats the first combination in the table, and then we add 2+1, which is also a repetition. Therefore, 1+1 and 2+1 are the only combinations which can be made with numbers less than 3.

4 can only be obtained for a sum by adding 3+1, 1+3, 2+2, 1+1+1+1, 2+1+1, 1+2+1, and 1+1+2. But, as 3+1 and 1+3 are essentially the same operation, only one of them need be placed in the table. To add 1+1+1+1, we first add 1+1, then 2+1, and then 3+1; to add 2+1+1, we add 2+1, and then 3+1; to add 1+2+1, we add 1+2, and then 3+1; and to add 1+1+2, we add 1+1, and then 2+2; none of which operations introduces any combinations not found in the table. By following this process with the higher numbers, it will be found that the above table contains all the combinations of the primitive numbers which can occur in addition.

But subtraction depends so directly on addition, that it seems impossible for a person to master one without mastering the other. Surely, no child can fully comprehend that 3 and 2 are 5, without as fully comprehending that 3 from 5 leaves 2, and that 2 from 5 leaves 3; for these are only different forms of expressing the idea that 5 is made up of 3 and 2. Why, then, should not addition and subtraction of the primitive numbers be so taught as to show this mutual dependence? The invariable experience of teachers who have given both methods a fair trial, has been that the two operations can be taught together much easier than either can be taught by itself.

These combinations mastered, their application to the decimal system is very easy. For example, since $3+2=5$, we have $13+2=15$, $23+2=25$, $83+2=85$, $193+2=195$, $30+20=50$, $300+200=500$, $3000+2000=5,000$, $30,000+20,000=50,000$, $53,000+2,000=55,000$, $34,000+20,000=54,000$, $63,000,000,000+2,000,000,000=65,000,000,000$, &c. &c.

Again, since $5-2=3$, $15-2=13$, $75-2=73$, $495-2=493$, $50-20=30$, $500-200=300$, $500,000-200,000=300,000$, $500,000,000-200,000,000=300,000,000$, &c. &c.

Such being the case, why should not the above table, with the reverse subtractions, and the corresponding table of combinations in multiplication and division, be thoroughly mastered by the pupil before he is allowed to pass on to higher numbers? And, when he passes to them, why should they not be so presented as to show their dependence on what he has previously learned?

A child will easily understand and perform a problem involving small numbers, when he will not be able to understand a similar one involving large numbers; or to perform it, except by some arbitrary rule learned only by rote. In one case, he will be able by a reasoning process of his own to obtain a result and test its truth; while in the other, he can neither reason on the conditions of the question, nor tell whether he has obtained a correct answer, except by reference to the printed answer in the text-book, or key. Can there be any doubt which of these two classes of questions will be the most valuable to the young pupil? Can there be any doubt that he should be drilled on problems involving small numbers, till his reasoning powers are somewhat developed, and he has acquired sufficient knowledge of the nature of numbers and numerical operations to enable him to use large numbers understandingly? We think there can be none; and hence, in the preparation of the Pestalozzian Series, we have endeavored so to arrange the exercises as to lead the pupil, by a gradual and systematic course of training, to a thorough knowledge of the nature and use of numbers, and numerical operations, and to a perfect command over them in all their varied combinations.

The FIRST STEPS IN NUMBERS, designed as the beginners' first book, is believed to be a thorough and completely elementary treatise. It deals entirely in small numbers, containing, in the first section, none higher than 5, in the second, none higher than 7, in the third, none higher than 9, &c.,

till in the seventh it contains them as high as 20. This, by keeping the numbers entirely within the comprehension of the learner, and by rendering it easy to give illustrations of the processes, and tests of the correctness of the work, makes it practicable to give for solution questions more complicated in their conditions than could otherwise be done, and secures that power of reasoning and just self-confidence, which must be the basis of all true mathematical attainments.

It presents the combinations of primitive numbers in a great variety of forms, so arranged as to show the relation of addition, subtraction, multiplication, and division to each other, and make the pupil familiar with the work which lies at the foundation of the art of arithmetic, and is as essential to it as is a knowledge of the alphabet to the art of reading.

It illustrates and explains all fractional operations.

It contains a great variety of practical problems, based on such incidents as are within the comprehension of the young learner, and likely to interest him, while they require attention and thought in their solution.

PART TWO commences where PART ONE ends, and involves larger numbers, and operations in "Written" as well as "Mental Arithmetic."

It illustrates fully the nature and simplicity of the Decimal System of representing numbers, and the methods of reckoning by it.

It contains a great variety of questions on abstract numbers, so arranged as to indicate the best methods of performing the operations which they require. The pupil should study these and practise upon them, till he can add, subtract, multiply and divide, by glancing from figure to figure, as he reads by glancing from word to word. The accountant reckons in this manner, and why should not the pupil learn to do it while in school? To insist upon this will, in the end, secure the best results, as it will make after progress rapid and sure. Moreover, no person can be fitted for the higher departments of arithmetic, or for active business, till he has so far mastered these mechanical operations as to be able to perform them without fear of error.

It contains a great variety of practical problems, many of which are arranged somewhat in the form of connected stories. This feature will make them of a more interesting character to the pupil, and more like the problems of real life, while it will render a mere mechanical solution impossible.

Instead of giving answers to each problem, it leaves the pupil to determine for himself the truth of his results, by carefully revising his work, or by performing it in some different way from that which he first adopted. As these are the only tests which can be used in real life, it seems absurd to place in a text-book, designed to be a *practical* treatise, that which makes their application entirely unnecessary to the student. Were such tests always required, and were the pupil taught from the outset to regard an error committed in the school-room as a thing of as much consequence as one committed in the counting-room, the careless habits which usually so hinder his progress would never be formed.

It recognizes no difference between the principles of "Mental" and "Written Arithmetic," but retains in the latter the peculiarities which give to the former its chief value.

It gives no arbitrary rules to make a mere machine of the pupil, but rather states and explains principles, leaving him to reason on them and apply them in such ways as his ingenuity suggests.

It explains Whole Numbers in connection with Decimal Fractions, and Simple Numbers with Compound. This seems far more philosophical and simple than it would be to treat them separately, for there is no principle involved in one that is not involved in the other.

Questions in Reduction, being of the same character with other questions in Multiplication and Division, need no separate treatment. It certainly requires no different reasoning process or numerical operation to tell *how many quarts there are in three gallons*, from that required to tell *how much three apples will cost at four cents apiece;* or to tell *how many weeks are equal to twenty-eight days*, from that required to tell *how many oranges at seven cents apiece can be bought for twenty-eight cents.*

Important contractions and abbreviated methods are contained in various parts of the work, being mostly explained in connection with the processes to which they apply. The last section, however, is devoted entirely to them.

A more full view of the work will be found in the annexed analysis of its contents.

Part Three, which is to be a higher arithmetic, will probably be published the ensuing Spring.

## ANALYSIS OF THE CONTENTS OF THIS WORK.

SECT. I. A.—Analysis of tens from twenty to one hundred, with methods of representing them. B.—Questions on A. C, D, E, and F.—Application of the decimal system to the addition and subtraction of *tens.* G.—Practical examples.

SECT. II. A.—Analysis of tens and units to one hundred, with methods of representing them. B and C.—Questions for practice on the principles explained in A. D, E, and F.—Application of the decimal system to additions and subtractions *depending on numbers less than ten.* Practical problems.

SECT. III. A, B, C, D, E, F, G, H, and I.—A great variety of questions to illustrate and make familiar the application of the decimal system to additions and subtractions *depending on any combination of the primitive numbers.* J. — Practical problems.

SECT. IV. This section involves such numbers below one thousand, as can be expressed by hundreds and tens. A. — Analysis of hundreds, with methods of representing them. B. — Questions on principles explained in A. C. — Analysis of hundreds and tens, with methods of representing them. D. — Questions on principles explained in C. E, F, G, H, and I. — A variety of questions to secure familiarity, with application of the decimal system to addition and subtraction *of numbers containing only hundreds and tens.* I. — Practical problems.

SECT. V. Numbers below one thousand. A. — Analysis of hundreds, tens and units, with methods of representing them. B, C, and D. — Questions on principles explained in A. E, F, and G. — Application of the decimal system to additions and subtractions depending on any combination of the primitive numbers. *These questions are like those in* SECT. III., *except that they involve numbers above one hundred.* H. — Practical examples.

SECT. VI. P. — Addition and subtraction of hundreds, tens, and units. A. — Addition of *tens* to hundreds, tens, and units. B. — Subtraction of *tens* from hundreds, tens, and units. C. — Addition of *tens and units* to hundreds, tens, and units. D — Subtraction of *tens and units* from hundreds, tens, and units. E and F. — Exercises for further practice in the addition and subtraction of numbers of two figures each. G, H, and I. — Addition and subtraction of numbers composed of hundreds, tens, and units. J. — Practical problems.

SECT. VII. A to Q. — Tables of money, weight, and measure. Q, to end of section. — Addition and subtraction of denominate numbers, with the reductions they involve. *Several of the solutions here given render the work much more easy than the methods ordinarily given in school text-books*, e. g. SEE Ex. 11, R. Ex. 14, S. Ex. 11, *and* Ex. 20, T.

SECT. VIII. Multiplication and division of primitive numbers, involving no products or dividends greater than twenty.

SECT. IX. Fractions. A, B, and C. — Explanations and illustrations of the nature of fractions, and the methods of representing them. D, E, and F. — Addition and subtraction of fractions. *For easy solutions similar to those referred to in Sect. 7th,* SEE Ex. 8 *and* 21, E, *and* Ex. 4 *and* 10, F. G. — Practical examples.

SECT. X. A, B, and C. — Multiplication of primitive numbers, with reverse division. D. — Application of multiplication to the reduction of whole and mixed numbers to improper fractions. *These questions require the simple but important reasoning process required by most questions in multiplication.* E. — Application of division to the reduction of improper fractions to whole numbers. *These questions require the application of the reasoning process required in a large class of questions in division.* F.—Multiplication and division of fractional quantities. G and H.—Miscellaneous exercises designed to secure accuracy and rapidity in numerical operations. I.—Practical examples in multiplication, including "Reduction Descending," and requiring application of the reasoning process given in D. J.—Practical examples in division, including "Reduction Ascending," and requiring application of the reasoning process given in E. K.—Practical examples in multiplication and division, requiring application of both reasoning processes.

Sect. XI. A.—Introductory to B. B and C.—Exercises in getting fractional parts of numbers. D.—Ratio, and applications of it. E.—Practical problems requiring a new form of reasoning.

Sect. XII. A.—Introductory to B. B.—Divisions giving a remainder. C, D, and E.—Application of preceding to various operations on fractions. F.—Practical problems requiring reasoning processes of Sect. X.

Sect. XIII. A, B, C, and D.—Application of principles of last section in getting fractional parts of numbers. E.—Practical problems, requiring application of reasoning processes of Sects. X. and XI. to reduction of Fractions of one Denomination to Whole Numbers of a Lower, and to Partnership, &c.

Sect. XIV. A, B, C, and D.—Application of previous reasoning processes in finding a number from some given fractional part of it. E.—Practical problems.

Sect. XV. A and B.—Application of the decimal system to the multiplication of *tens* by *units*. C and D.—Divisions, reverse to the above multiplications. E, F, and G.—Exercises to secure accuracy and rapidity in the previous operations. H.—Practical problems.

Sect. XVI. A.—Multiplication of numbers composed of *tens and units*. B.—Divisions, reverse operations to those in last letter. C and D.—Application to fractional operations. E and F.—Multiplication and division of numbers composed of hundreds, tens, and units. G.—Application to "Reduction Descending and Ascending." H.—Practical problems.

Sect. XVII. A.—Definitions and illustrations of powers and roots. B.—Questions for exercise in finding powers and roots of numbers. C.—Practical problems in Mensuration.

Sect. XVIII. Federal Money. A and B.—Reductions. C.—Methods of representing values by figures, with illustrations of effect of changing the place of the decimal point. D and E.—Exercises in writing and reading numbers. F.—Practical problems. G.—Exercises on the coins of the United States. H.—Exercises in finding convenient coins for the payment of specified sums. I.—Exercises in "making change."

*The problems in the last three letters are highly practical, and very interesting to pupils, while they afford excellent mental discipline.*

SECT. XIX. NOTATION AND NUMERATION. Introductory Note. Comparison of Oral with Written Arithmetic. A. — The digits and their uses as previously illustrated. B. — Places and periods used in writing large numbers. C. — Double use of figures, viz: to mark places, and determine number of units of each denomination, with exercises in writing them. D and E. — Exercises in reading and writing large numbers. F and G. — Further analyses of numbers, with different methods of reading them, *designed to test the accuracy of the pupil's knowledge, and his skill in applying it.* H. — Ratio of the values of a figure as determined by its place. I. — Extension of principles to places at the right of the point, or to those used in writing decimal fractions. J. — Methods of reading decimal fractions, with examples for practice. K. — Exercises in writing decimal fractions. L. — Change of point in multiplying and dividing by powers of ten. M. — Reduction of vulgar fractions to equivalent decimals.

SECT. XX. ADDITION. A. — Definitions and illustrations. B. — Methods illustrated with both simple and compound numbers. C. — Methods of proof. D. — Best method with compound numbers. Examples for practice.

SECT. XXI. SUBTRACTION. A. — Definitions and illustrations. B. — Method when no reductions are necessary. C. — Methods of proof. D. — Examples for practice. E. — Reductions and changes in minuend, with examples for practice. F. — Reductions made before beginning to subtract. G. — Reductions not written out. Best method with compound numbers. Problems. H. — Method by beginning at the left. I. — Method of subtracting several numbers at one operation, with practical problems. *Such reductions as are required in* Ex. 6, 8, 12, *&c.*, F, *and* Ex. 17, G, *are peculiarly valuable to exercise the ingenuity of the pupil, and test his skill.*

SECT. XXII. MULTIPLICATION. A. — Definitions and illustrations. B. — Demonstration that product is not affected by change in order of factors. C. — Method when multiplier is expressed by a single figure, multiplicand being either a simple number, decimal fraction, or compound number. D. — Methods of proof. E. — Examples for practice, and practical problems. *Improved processes are suggested in connection with* Ex. 15 *and* 23. *Change of place of point, when the multiplier is a decimal fraction, is illustrated in solution to* Ex. 25, 45, *and* 46, *as are Commission and Interest in* Ex. 37 *and* 40, *and the rules following them.* F. — Multiplication by factors. G. — Method when multiplier is expressed by more than one figure. H. — Multiplication by two or more figures at one operation. Practical problems.

SECT. XXIII. DIVISION. A. — Definitions and illustrations. Two classes of Questions. B. — Method of dividing when divisor is a small number.

Proof. First class of questions. C. — The same with second class of questions. Application to problems involving fractional operations, and compound numbers. D. — Change of point when the divisor is a decimal fraction. E. — Division by factors, with method of getting the true remainder. F. — Recapitulation of principles. Applications when divisor is a large number. Use of trial divisor. *These exercises will probably anticipate or remove the difficulties which the pupil usually finds in "Long Division."* G. — "Long Division." Practical problems. *An explanation of an abbreviated process follows* Ex. 16.

SECT. XXIV. CONTRACTIONS AND SHORT PROCESSES. A. — Addition. B. — Subtraction. C. — Multiplication by 99, 999, &c. D. — Multiplication by factors of 10, 100, 1000, &c. E. — Multiplication when one part of the multiplier is a factor of another part. F. — Division by 9, 99, 999, &c. G. — Division when divisor is a factor of some power of 10. H. — Multiplication of numbers whose tens are alike. Squaring of numbers. I. — Multiplication of numbers whose units are alike. J. — Multiplication of the *sum of two numbers by their difference.* K. — Squaring of numbers ending in 25 and 75. L. — Other applications of preceding contractions. M. — Contractions in multiplying several numbers together. N. — Contractions when sum of several products is required. O. — The old rule of "Practice."

# EXPLANATION OF SIGNS.

THERE are many characters used to indicate arithmetical operations, among which are the following: —

$=$ The sign of equality, called *equals* or *is equal to*, signifies that the quantities between which it is placed are in value equal to each other.

$+$ The sign of addition, called *plus* or *and*, signifies that the numbers between which it is placed are to be added together. Thus $7 + 4 = 11$ means that 7 added to 4 are equal to 11. It may be read either 7 *plus* 4 *equals* 11, or 7 *and* 4 *are* 11.

— The sign of subtraction, called *minus* or *less*, signifies that the number following it is to be subtracted. Thus $9 - 6$ means that 6 is to be subtracted from 9, and may be read 9 *minus* 6, or 9 *less* 6.

$\times$ The sign of multiplication, called *times*, or *multiplied by*, signifies that the quantities between which it is placed are to be multiplied together. $7 \times 6$ means that 7 and 6 are to be multiplied together, and may be read either as 7 *times* 6, or 7 *multiplied by* 6.

$\div$ The sign of division, called *divided by*, signifies that the quantity preceding it is to be divided by that which follows it. $12 \div 3$ means that 12 is to be divided by 3. It may be read as 12 *divided by* 3.

The parenthesis ( ) is used to enclose several quantities connected by the preceding signs, and to show that the *result* of the operations indicated within the parenthesis is to be regarded as the quantity affected by the signs preceding or following the parenthesis. Thus $6 \times (2 + 4)$ signifies that 6 times the sum of 2 and 4 is to be obtained. Written without the parenthesis, thus, $6 \times 2 + 4$, it would mean that to 6 times 2, 4 is to be added.

The vinculum, a straight line drawn above the written figures and signs, is often used in place of the parenthesis, and has the same use. Thus $6 \times \overline{2 + 4}$ has the same meaning as $6 \times (2 + 4)$.

In this work the comma is used in connection with the above signs, to show that the *result* of *all* the indicated operations preceding it is to be considered in connection with the signs following it.

$6 \times 8, \div 4, \times 3 + 7 \times 2$, signifies that 6 times 8, or 48, is to be divided by 4; that this result, 12, is to be multiplied by 3, and that to 36, the result thus obtained, 7 times 2, or 14, is to be added. The final result is therefore 50.

$6 \times 8, \div 4 \times 3 + 7 \times 2$, signifies that 6 times 8 is to be divided by 4 times 3, and that 7 times 2 is to be added to this quotient. The result is therefore 18.

$6 \times 8, \div 4, \times 3 + 7, \times 2$, signifies that 6 times 8 is to be divided by 4, and this quotient multiplied by 3; that 7 is to be added to this result, and that the sum thus obtained is to be multiplied by 2. The final result is therefore 86.

We have also used commas in connection with figures and signs to indicate ellipses. For instance, in explaining the addition of $7 + 8 + 3 + 9$, we have written $8 + 7 = 15, + 3 = 18, + 9 = 27$, as a convenient way of expressing that $8 + 7 = 15$, that $15 + 3 = 18$, and that $18 + 9 = 27$.

# THE

# DECIMAL SYSTEM OF NUMBERS.

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## SECTION I.

**A.** 1. 10 + 10 = 2 tens, called TWENTY, written 20.
2. 2 tens + 10 = 3 tens, called THIRTY, written 30.
3. 3 tens + 10 = 4 tens, called FORTY, written 40.
4. 4 tens + 10 = 5 tens, called FIFTY, written 50.
5. 5 tens + 10 = 6 tens, called SIXTY, written 60.
6. 6 tens + 10 = 7 tens, called SEVENTY, written 70.
7. 7 tens + 10 = 8 tens, called EIGHTY, written 80.
8. 8 tens + 10 = 9 tens, called NINETY, written 90.
9. 9 tens + 10 = 10 tens, called ONE HUNDRED, written 100.

NOTE. The period at the right of the above written numbers (20., 30., 40., &c.), is called the *decimal point*, and is used to denote the place of ones, or units. The first place at the left of the decimal point is the place of units, the second place is that of tens, and the third, that of hundreds. When the decimal point is omitted, it is understood to belong at the right of the figures representing the number.

**B.** 1. What do we call 8 tens? 9 tens? 6 tens? 7 tens? 4 tens? 2 tens? 5 tens? 3 tens? 10 tens?

2. How is twenty written by figures? Sixty? Forty? Ninety? Eighty? Thirty? Fifty? Seventy? One hundred?

3. How many tens in 50? 80? 20? 90? 100? 70? 30? 60? 40? 10?

**C.** 1. 4 tens + 4 tens = how many tens? Called what? How many then are 40 + 40?

2. 7 tens + 2 tens = how many tens? Called what? How many then are 70 + 20?

3. 5 tens + 3 tens = how many tens? Called what? How many then are 50 + 30?

Let the pupil now read the following, supplying appropriate numbers in place of the stars.

4. 2 tens + 5 tens = * tens, or * units; then 20 + 50 = *.
5. 4 tens + 5 tens = * tens, or * units; then 40 + 50 = *.
6. 3 tens + 6 tens = * tens, or * units; then 30 + 60 = *.
7. 2 tens + 2 tens = * tens, or * units; then 20 + 20 = *.
8. 3 tens + 7 tens = * tens, or * units; then 30 + 70 = *.
9. 3 tens + 3 tens = * tens, or * units; then 30 + 30 = *.
10. 6 tens + 4 tens = * tens, or * units; then 60 + 40 = *.
11. 2 tens + 3 tens + 4 tens = * tens, or * units; then 20 + 30 + 40 = *.
12. 5 tens + 2 tens + 3 tens = * tens, or * units; then 50 + 20 + 30 = *.
13. 3 tens + 2 tens + 3 tens = * tens, or * units; then 30 + 20 + 30 = *.
14. 5 tens + 1 ten + 2 tens = * tens, or * units; then 50 + 10 + 20 = *.
15. 2 tens + 1 ten + 3 tens = * tens, or * units; then 20 + 10 + 30 = *.

**D.** 1. 9 tens — 6 tens = how many tens? How many units? 90 — 60?

2. 7 tens — 3 tens = how many tens? How many units? 70 — 30?

3. 5 tens — 4 tens = how many tens? How many units? 50 — 40?

Read the following, supplying the proper numbers in place of the stars.

4. 8 tens — 5 tens = * tens, or * units; then 80 — 50 = *.
5. 9 tens — 7 tens = * tens, or * units; then 90 — 70 = *.
6. 6 tens — 3 tens = * tens, or * units; then 60 — 30 = *.

7. 10 tens — 2 tens = * tens, or * units; then 100 — 20 = *.

8. 7 tens — 5 tens = * tens, or * units; then 70 — 50 = *.

9. 8 tens — 4 tens = * tens, or * units; then 80 — 40 = *.

10. 10 tens — 5 tens = * tens, or * units; then 100 — 50 = *.

11. 10 tens — 3 tens — 4 tens, = * tens, or * units; then 100 — 30 — 40 = *.

12. 6 tens — 3 tens — 2 tens, = * tens, or * units; then 60 — 30 — 20 = *.

13. 8 tens — 3 tens — 3 tens, = * tens, or * units; then 80 — 30 — 30 = *.

14. 9 tens — 2 tens — 2 tens, = * tens, or * units; then 90 — 20 — 20 = *.

15. 10 tens — 5 tens — 4 tens, = * tens, or * units; then 100 — 50 — 40 = *.

**E.** 1. 30 + 50? Solution, 30 + 50 = 3 tens + 5 tens = 8 tens, or 80. Ans.

2. 40 + 50? 30 + 70? 40 + 20? 60 + 10?

3. 20 + 70? 50 + 20 + 20? 30 + 20 + 10 + 40?

4. 10 + 20 + 30 + 30? 20 + 40 + 20 + 10? 30 + 20 + 10 + 20 + 20?

5. 30 + 30 + 20? 10 + 20 + 50 + 20? 10 + 30 + 60?

6. 10 + 10 + 80? 10 + 60 + 20? 20 + 20 + 20 + 30?

NOTE. The solutions in the above examples, and also in the following, should be omitted as soon as the pupil has become so familiar with the operation as to answer the questions promptly, when the results simply of the successive combinations should be given; as in the following example. 20 + 20 + 20 + 30? 20, 40, 60, 90, Ans.

**F.** 1. 90 — 60? Solution, 90 — 60 = 9 tens — 6 tens = 3 tens, or 30. Ans.

2. 80 — 50? 60 — 30? 90 — 20? 50 — 10 — 20?

3. 70 — 40 — 20? 100 — 30 — 10? 20 + 40 + 40 — 60?

4. 50 + 40 — 30 — 40 + 60? 80 — 30 + 10 + 40 — 70?

5. 100 — 10 — 10 — 30 + 20 — 50 — 10? 10 + 10 + 10 + 40 + 10 — 40 — 20 — 10?

6. 90 — 60 + 20 — 10 + 60 — 20? 30 + 30 + 30 — 40 — 40 + 30 + 30 + 30 — 40?

7. 30 + 40 — 20 + 50 — 70 — 20? 80 — 40 — 20 + 70 + 10 — 30?

8. 100 — 20 — 20 — 20 — 20? 90 — 20 — 20 — 20 — 20?

9. 30 + 30 + 30 + 10 — 40 — 40 + 30? 50 + 40 — 30 — 20 — 10 + 20?

10. 70 — 20 + 40 — 60 — 20 + 70? 50 + 30 — 10 — 60 + 50 + 20?

**G.** 1. Mr. Phillips expended 40 dollars on an acre of potatoes, 30 dollars on one acre of corn, and 20 dollars on a half acre of corn. How much did he expend on all? How much more on the corn than on the potatoes? How much more on the potatoes than on the one acre of corn?

2. A bushel of potatoes weighs 40 pounds, a bushel of wheat weighs 60 pounds. What is the weight of both? How many more pounds of power would a person have to employ to lift a bushel of wheat, than to lift a bushel of potatoes?

3. James carried a bushel of beans and a bushel of potatoes to Mrs. Goss. What was the weight of his load, allowing the weight of beans and wheat to be equal?

4. A horse ate 10 pounds of hay from a truss weighing 60 pounds, and a yoke of oxen ate 30 pounds from the same truss. How many pounds were eaten? How many were left?

5. Esquire Jones bought for his farm a plough-horse worth 50 dollars, a plough worth 10 dollars, and an ox cart worth 20 dollars. What was the value of all? How much more was the horse worth than the plough? Than the cart? Than the plough and cart together? He afterwards bought a carriage-horse for 100 dollars. How much more did he pay for it than for his plough-horse? How much more than for his plough-horse, plough, and cart? He sold his plough-horse at a cattle-show for 60 dollars, and paid 40 dollars of what he received, for a yoke of steers, and the rest for a cow. How much did he pay for the cow? How much money did he gain upon his horse?

6. Mr. Tyler had in his farm 10 acres of wheat, 10 of rye, 10 of corn, and 50 of pasturage. How many acres had he in all? How many more acres of pasturage had he than of wheat, rye, and corn? He exchanged his wheat-field for 20 acres of woodland. How many acres had he then in his farm? What was the difference between the number of acres in his piece of woodland, and the number of those in his rye and corn fields taken together? 40 acres of his pasturage, and 10 of his wood-

land, were enclosed by a stone wall, and the rest of his land by cedar fences. How many acres were fenced with cedar? He kept 10 cows in one pasture, and 20 in another, and 10 calves in a third. Beside these, he had, reckoning his horses, oxen, pigs and sheep, 20 quadrupeds,* and he had 30 hens, and 60 chickens. How many quadrupeds had he? How many bipeds?† How many more bipeds than quadrupeds?

7. Mr. Smith had in his orchard 30 apple-trees, 40 peach-trees, 20 plum-trees, and 10 pear-trees. How many trees had he in all? During a severe winter, 20 of his peach-trees and 10 of his plum-trees were destroyed. How many trees had he left? The next fall he sold 60 bushels of peaches, and 30 of pears, at a dollar a bushel. What did he receive for the whole? He gathered 40 barrels of baldwins, 10 of Roxbury russets, 30 of greenings, and 20 of pippins. How many barrels of apples did he gather? He reserved 3 barrels of baldwins, 1 of russets, 3 of greenings, and 3 of pippins; the rest he sold. How many did he reserve? How many did he sell?

8. Mr. Wilson went a journey of 100 miles in a carriage. The first day he rode 30 miles, and the second day 40. How many more miles had he to ride? On his return he rode 40 miles the first day. How much further had he to ride? He met 50 carriages on his outward journey, and 40 on his return. How many did he meet in going and returning? How many more in going than in returning?

9. A farmer, who owned 90 sheep, kept 40 in one pasture, 30 in another, and the rest in another. How many did he keep in the last pasture? He drove 10 sheep from the second pasture into the first, and the rest of those in the second pasture he drove into the third. How many were there in the first pasture? In the third? How many more were there in the first pasture than in the third? If he should now drive into the second pasture 30 sheep from the first pasture, and 20 from the third, how many would there be in each pasture?

10. Sarah found 20 walnuts under one tree, and 30 under another, while Lydia found 30 under one tree, and 20 under another. How many did each find? did both? Sarah gave 20 nuts to Jane, and Lydia gave her 10. How many did both give her? How many had each left? How many did the three girls together have? Sarah gave 10 nuts to Fanny, Lydia gave 20, and Jane gave 10. How many did all give her? How many did each have left? How many nuts did the four girls together have?

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* A quadruped is an animal having four feet.

† A biped is an animal having two feet.

## SECTION II.

**A.**

1. 2 tens + 1 unit = 20 + 1, or TWENTY-ONE, written 21.
2. 2 tens + 2 units = 20 + 2, or TWENTY-TWO, written 22.
3. 2 tens + 3 units = 20 + 3, or TWENTY-THREE, written 23.
4. 2 tens + 4 units = 20 + 4, or TWENTY-FOUR, written 24.
5. 2 tens + 5 units = 20 + 5, or TWENTY-FIVE, written 25.
6. 2 tens + 6 units = 20 + 6, or TWENTY-SIX, written 26.
7. 2 tens + 7 units = 20 + 7, or TWENTY-SEVEN, written 27.
8. 2 tens + 8 units = 20 + 8, or TWENTY-EIGHT, written 28.
9. 2 tens + 9 units = 20 + 9, or TWENTY-NINE, written 29.
10. 3 tens + 1 unit = 30 + 1, or THIRTY-ONE, written 31.
11. 3 tens + 7 units = 30 + 7, or THIRTY-SEVEN, written 37.
12. 4 tens + 7 units = 40 + 7, or FORTY-SEVEN, written 47.
13. 6 tens + 8 units = 60 + 8, or SIXTY-EIGHT, written 68.
14. 7 tens + 3 units = 70 + 3, or SEVENTY-THREE, written 73.
15. 8 tens + 9 units = 80 + 9, or EIGHTY-NINE, written 89.
16. 5 tens + 6 units = 50 + 6, or FIFTY-SIX, written 56.
17. 9 tens + 5 units = 90 + 5, or NINETY-FIVE, written 95.

**B.** 1. What do we call 70 + 8? 30 + 7? 90 + 9?
2. 50 + 1? 40 + 9? 80 + 6? 70 + 3? 60 + 7?
3. 30 + 7? 50 + 9? 20 + 5? 60 + 2? 70 + 7?
4. What do we call 5 tens and 8 units?
5. 6 tens and 3 units? 8 tens and 4 units?
6. 9 tens and 9 units? 3 tens and 9 units?
7. 7 tens and 1 unit? 6 tens and 7 units?
8. 8 tens and 7 units? 9 tens and 4 units?
9, 3 tens and 3 units? 5 tens and 0 units?
10. How will each of the above values be represented by figures?

**C.** 1. What number will be represented by 7 written in the tens' and 5 in the units' place?
2. By 6 in the tens' and 4 in the units'?
3. By 8 in the tens' and 9 in the units'?

4. By 5 in the tens' and 7 in the units'?
5. By 9 in the tens' and 0 in the units'?
6. By 8 in the tens' and 3 in the units'?
7. By 0 in the tens' and 9 in the units'?
8. By 8 in the tens' and 8 in the units'?
9. By 5 in the tens' and 0 in the units'?
10. How many tens and units in 64?
11. In 73? in 85? in 97? in 60? in 15?
12. In 03? in 07? in 82? in 80? in 30?
13. In 09? in 05? in 50? in 04? in 40?
14. In 24? in 42? in 75? in 57? in 49?
15. In 94? in 83? in 67? in 76? in 99?

**D.** 1. 4 + 2 + 2? 40 + 20 + 20? 14 + 2 + 2?
2. 24 + 2 + 2? 34 + 2 + 2? 64 + 2 + 2?
3. 94 + 2 + 2? 4 + 22 + 2? 4 + 52 + 2?
4. 4 + 72 + 2? 4 + 2 + 92? 4 + 2 + 42?
5. 2 + 5 + 2? 20 + 50 + 20? 12 + 5 + 2?
6. 22 + 5 + 2? 92 + 5 + 2? 2 + 35 + 2?
7. 2 + 65 + 2? 2 + 5 + 72? 2 + 5 + 12?
8. 3 + 4 + 3? 30 + 40 + 30? 13 + 4 + 3?
9. 23 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3?
10. 2 + 3 + 5 + 2 + 3 + 5 + 30 + 2 + 3 + 5 + 2 + 3 + 5 + 10 + 2 + 3 + 5?
11. 4 + 5 + 1 + 4 + 5 + 1 + 4 + 5 + 1 + 4 + 5 + 1 + 50 + 4 + 5 + 1?
12. 2 + 5 + 3 + 20 + 2 + 5 + 3 + 2 + 5 + 3 + 2 + 5 + 3 + 2 + 5 + 3 + 2?
13. 3 + 4 + 3 + 2 + 5 + 3 + 3 + 4 + 3 + 2 + 5 + 3 + 40 + 3 + 4 + 3 + 2 + 5?
14. 2 + 3 + 3 + 2 + 4 + 6 + 5 + 5 + 7 + 3 + 2 + 1 + 3 + 1 + 2 + 1 + 3 + 7 + 4 + 2?
15. 23 + 2 + 5 + 4 + 4 + 2 + 7 + 3 + 2 + 1 + 5 + 2 + 1 + 9 + 10 + 2 + 7?

**E.** 1. 9 — 3 — 4? 90 — 30 — 40? 19 — 3 — 4?

2. 29 — 3 — 4? 69 — 3 — 4? 89 — 3 — 4?
3. 99 — 3 — 4? 79 — 3 — 4? 39 — 3 — 4?
4. 8 — 3 — 2? 80 — 30 — 20? 18 — 3 — 2?
5. 38 — 3 — 2? 98 — 3 — 2? 58 — 3 — 2?
6. 10 — 2 — 5 — 3? 100 — 20 — 50 — 30?
7. 20 — 2 — 5 — 3? 40 — 2 — 5 — 3?
8. 100 — 2 — 5 — 3 — 2 — 5 — 3 — 2 — 5 — 3?
9. 10 — 3 — 4 — 3? 100 — 30 — 40 — 30?
10. 60 — 3 — 4 — 3 — 3 — 4 — 3 — 2 — 5 — 3 — 3 — 4 — 3 — 2?
11. 10 — 2 — 6 — 2?
12. 100 — 2 — 6 — 2 — 2 — 6 — 2 — 3 — 4 — 3 — 2 — 5 — 3 — 2 — 6 — 2 — 3 — 4 — 3 — 2 — 6?
13. 8 — 5 — 3?
14. 98 — 5 — 3 — 2 — 5 — 3 — 2 — 6 — 2 — 3 — 4 — 3 — 2 — 5 — 3 — 3?
15. 10 — 7 — 1 — 2?
16. 50 — 7 — 1 — 2 — 7 — 1 — 2 — 7 — 3 — 5 — 5 — 4 — 2 — 4?

**F.** 1. 5 + 4 + 8 + 3 + 6 + 4 + 40 + 3 + 5 — 8 — 3 — 2 — 5 — 7 + 4 + 3 + 8 — 2?
2. 24 + 6 + 8 — 3 + 5 — 7 — 3 + 20 — 6 — 2 + 8 + 7 + 3 — 2 — 5 — 3 + 20?
3. 19 + 1 + 5 + 4 — 8 + 9 + 20 — 10 + 3 + 7 + 2 + 8 — 30 + 7 + 3 + 50 — 20?
4. 87 — 3 — 4 — 6 — 4 — 30 + 2 + 5 — 6 + 9 + 10 — 3 — 5 + 8 + 20 — 6?
5. 70 + 3 + 7 + 5 + 5 + 9 + 1 — 30 — 20 — 6 — 2 — 2 — 2 — 8 — 1 — 3 — 3 — 3 — 3 — 3 — 3?
6. 53 + 3 + 4 + 10 — 3 — 2 — 4 + 1 + 8 + 30 — 5 — 5 — 7 — 3 — 2 — 6 — 2 — 3 — 7 — 4?
7. 19 + 1 + 50 — 20 + 6 + 4 + 7 + 3 + 2 + 8 + 1 + 9 — 6 — 4 — 1 — 9 — 3 — 7 — 2 — 8 + 5?

**G.** 1. In a primary school 30 pupils occupied the first row of chairs, 20 the second row, and 9 the third. How many were there in all? How many more were there in the first row than in the second? than in the third? How many more in the first and second than in the third?

2. In a certain school, 12 pupils were in the first class, 8 in the second, 20 in the third, and 9 in the fourth. How many belonged to the school? How many more were in the third class than in each of the others?

3. The pupils of a certain school, having contributed the sum of 20 dollars towards improving their school library, received 3 dollars from Mr. Jenks, 7 dollars from Mr. Jones, and 20 dollars from a wealthy gentleman of Boston. Mr. Cross gave 3 dollars' worth of books, Mr. Thompson 7 dollars' worth, Mr. Jenkins 2 dollars' worth, Mr. Fletcher 4 dollars' worth, Mrs. Brown 3 dollars' worth, and Mr. Williams 1 dollar's worth, after which Mrs. Hale gave 6 dollars in cash, and Mr. Stearns 4 dollars. What was the value of the books and money? What was the value of the books alone? How much money had they to expend for books after reserving 9 dollars for a book-case?

4. James Homer, Samuel Capen, and Robert Greene, for disturbing the good order of a school, were fined as follows: — Homer 30 dollars, Capen 30 dollars, Greene 20 dollars. What was the amount of their fines? James Homer's father wished to pay his son's fine, but as he had but 10 dollars in ready money, he sold his great coat for 8 dollars, his best hat for 2 dollars, and borrowed the rest from his employer. How many dollars did he borrow? Capen and Greene, from inability to pay, would have been sent to the county jail, Capen for 40 days and Greene for 30, had not their teacher paid their fines. How many days would have been lost by both had they been imprisoned? In order to obtain money to repay the teacher, Samuel worked 30 days for 14 dollars, 10 days for 4 dollars, 6 days for 2 dollars, 4 days for 3 dollars, 7 days for 4 dollars, 3 days for 1 dollar, and 7 days for the remainder. How many days did he have to labor? How many dollars did he receive for the last 7 days' labor? For the same purpose Robert worked 15 days for 5 dollars, 5 days for 2 dollars, 7 days for 3 dollars, 3 days for 1 dollar, 8 days for 3 dollars, 2 days for 1 dollar, and 10 days for the remainder of the debt. How many days did he have to labor? How many dollars did he receive for the last 10 days' labor?

5. In a high school there were 11 errors in spelling sacrifice, 9 in privilege, 5 in chimneys, 4 in inflammatory, 1 in amphitheatre, 3 in wholly, 7 in fiery, 6 in coercion, 4 in believe, 2 in divide, 1 in benefit, 7 in sacrilegious, 4 in preferred, 3 in disciple. How many errors were committed?

6. The following numbers were given in school to be added: 2, 3, 5, 7, 3, 1, 8, 1, 1, 4, 5, 3, 3, 4, 8, 2, 6, 3, 1, 7, 3, 1, 8, 1, 3, 2, 1. 20 pupils gave 95 for the sum, 30 gave 96, 3 gave 98, 4 gave 86, 3 gave 92, and the rest of the scholars, 20 in number, gave no answer. How many were wrong, reckoning as wrong those who gave no answer? How many scholars were there in the school?

7. A trader bought some molasses for 20 dollars, some sugar for 10 dollars, some butter for 6 dollars, some cheese for 4 dollars, some flour for 30 dollars, and then had 5 dollars left. How many dollars did he have at first? He sold 7 gallons of molasses to one man, 3 to another, 5 to another, 5 to another, 10 to another, 2 to another, 7 to another, 1 to another, 3 to another, 7 to another, 1 to another, 9 to another, 20 to another, and 4 to another. How many gallons of molasses did he sell? He sold at one time 17 pounds of sugar, at another 3 pounds, at another 4 pounds, at another 4 pounds, at another 2 pounds, at another 1 pound, at another 9 pounds, at another 20 pounds, at another 2 pounds, at another 3 pounds; at another 3 pounds, at another 2 pounds, and at another 3 pounds, which was all that he had. How many pounds of sugar had he? The butter which he bought weighed just 35 pounds; he gave away 5 pounds of it, sold 20 pounds, and kept the remainder for his own use. How much did he keep for his own use? He sold 6 pounds of cheese to one man, 7 to another, 7 to another, 1 to another, 3 to another, 4 to another, 2 to another, 6 to another, 4 to another, and had 6 pounds left. How many pounds of cheese did he have at first?

8. Sarah has 4 picture books, Jane has 6, Abby has 8, Susan has 2, and Mary has 5. How many have they all? In one of Sarah's books there are 10 pictures, in another 5, in another 5, and in another 7. How many are there in all Sarah's books? In one of Abby's books are 20 pictures, in another 5, in another 2, in another 3, in another 10, in another 7, in another 3, and in another 4. How many in all Abby's books? Susan has 20 pictures in both her books, and Mary 30 pictures in all hers. How many have Susan and Mary together?

9. Daniel Parker bought a large lot of wood at auction, which he chopped himself. The first week he chopped 12 cords, the second 5, the third only 3, the fourth 8, the fifth 2, and the sixth 10. How many cords did he chop in all? The second week he sold 5 cords of wood, the third 7, the fourth 8, the fifth 0, and the sixth 9. How many cords did he sell? How many cords remained unsold at the close of each week?

10. A party went to a sleigh-ride. First there were 20 persons in a large four-horse sleigh, then 2 persons in one single sleigh, 4 in another, 2 in another, 2 in another, and 3 in another, then 7 in a double sleigh, 20 in a four-horse sleigh, and 30 in a six-horse sleigh. How many were there in the party? On reaching a fork in the road, the six-horse sleigh and the 2 four-horse sleighs took the right hand road, while the others took the left. How many persons took the right hand road? How many the left?

11. Mr. Hubbard went out to pay some debts that he owed, and collect some money that was due to him, taking 30 dollars with him. He paid 7 dollars to a shoemaker, 3 dollars to a laborer, and 5 dollars to a hatter: he then received 5 dollars from Mr. Golding, 30 dollars from Mr. Tubbs, 8 dollars from Mr. Evans, 2 dollars from Mr. Foster, 6 dollars from Mr. Crocker, and 3 dollars from Mr. Stone: after which he paid Mr. Guild 8 dollars for horse-hire, and 1 dollar for omnibus tickets, and Dr. Howe 7 dollars for services as a physician. How much money did he have left?

12. A boy once stood on Charlestown bridge, to see how many persons would ride over in half an hour. 16 persons passed in an omnibus, 4 in a covered wagon, 2 in a chaise, 2 in an open wagon, 2 in a rockaway, 3 on horseback, 1 in a gig, 4 in a cab, 6 in a hack, 9 in an omnibus, 1 on a loaded wagon, 8 in a stage coach, 2 in a chaise, and 7 in an omnibus. How many rode over in all?

13. Eliphalet and Timothy played "odd or even" together, beginning their games with 20 grains of corn apiece. The first trial Eliphalet won 3 grains from Timothy, the second 4, and the third 3; the fourth Timothy won 8, the fifth 2; and the sixth 3; the seventh Eliphalet won 3, and the eighth 9; the ninth Timothy won 7, the tenth 2, the eleventh 5, and the twelfth 4. How many grains did each boy have after the first trial? after the second? third? fourth? fifth? sixth? seventh? eighth? ninth? tenth? eleventh? twelfth?

14. Waldo was marked for his conduct and recitations in school during 4 weeks as follows, viz.: — On Monday of the first week, he received 6 merit-marks, on Tuesday 8, on Wednesday 6, on Thursday 7, but on Friday he lost 4 by his bad conduct, and school did not keep on Saturday. On Monday of the second week he received 7, and on Tuesday 8; on Wednesday he lost 4, and on Thursday 3; but on Friday he received 9. On Monday of the third week he lost 6, and on Tuesday 3; on Wednesday he received 4, on Thursday 5, and

on Friday 8. On Monday of the fourth week he lost 3; on Tuesday he received 5, on Wednesday 6, on Thursday 4, and on Friday 9. How many merit-marks did he have at the end of the four weeks? How many had he lost in the time?

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## SECTION III.

**A.** 1. 40 + 10 + 7? 10 + 7 + 40? 17 + 40? 40 + 17?
2. 80 + 10 + 7? 10 + 7 + 80? 17 + 80? 80 + 17?
3. 20 + 10 + 7? 10 + 7 + 20? 17 + 20? 20 + 17?
4. 30 + 17? 40 + 17? 50 + 17? 60 + 17? 70 + 17?
5. 20 + 10 + 5? 10 + 5 + 20? 15 + 20? 20 + 15?
6. 30 + 10 + 5? 10 + 5 + 30? 15 + 30? 30 + 15?
7. 50 + 15? 80 + 15? 15 + 50? 40 + 15? 70 + 15?
8. 60 + 10 + 9? 10 + 9 + 60? 19 + 60? 60 + 19?
9. 40 + 10 + 9? 9 + 10 + 40? 19 + 40? 40 + 19?
10. 20 + 19? 70 + 19? 19 + 30? 10 + 19? 80 + 19?
11. 30 + 14? 60 + 14? 50 + 13? 40 + 18? 16 + 70?
12. 40 + 12? 10 + 11? 60 + 11? 30 + 11? 10 + 12?
13. 70 + 11? 13 + 60? 18 + 20? 40 + 14? 50 + 14?
14. 20 + 17 + 3 + 17 + 3 + 17?
15. 40 + 19 + 1 + 19 + 1 + 14 + 6?
16. 60 + 11 + 6 + 3 + 11 + 6 + 3 — 30 + 11 + 6?
17. 80 + 16 — 3 — 3 — 50 + 16 + 4 + 16?
18. 10 + 14 + 6 + 14 — 4 + 14 — 4 + 14 + 2?
19. 20 + 13 + 7 + 13 + 7 + 13 — 3 + 13 — 3 + 13?
20. 50 + 18 — 8 + 12 + 8 + 16 — 3 + 4 — 7 — 20?
21. 30 + 11 + 9 + 11 — 1 + 12 — 2 + 17 + 3 + 8?

**B.** 1. 7 + 8 + 20? 20 + 7 + 8? 27 + 8? 8 + 27?
2. 7 + 8 + 40? 40 + 7 + 8? 47 + 8? 8 + 47?
3. 7 + 8 + 60? 60 + 7 + 8? 67 + 8?
4. 77 + 8? 8 + 77? 87 + 8? 37 + 8? 57 + 8?
5. 6 + 6 + 20? 20 + 6 + 6? 26 + 6? 6 + 26?
6. 6 + 6 + 80? 80 + 6 + 6? 86 + 6? 6 + 86?
7. 36 + 6? 56 + 6? 46 + 6? 6 + 62? 6 + 72?

8. 9 + 8 + 20? 20 + 9 + 8? 29 + 8? 8 + 29?
9. 69 + 8? 89 + 8? 59 + 8? 8 + 19? 8 + 39?
10. 5 + 8? 25 + 8? 75 + 8? 65 + 8? 35 + 8?
11. 8 + 85? 8 + 55? 8 + 15? 45 + 8? 85 + 8?
12. 7 + 4? 17 + 4? 87 + 4? 4 + 27? 4 + 57?
13. 4 + 37? 77 + 4? 67 + 4? 4 + 47? 47 + 4?
14. 8 + 7? 5 + 6? 8 + 7 + 6? 18 + 7 + 6?
15. 28 + 7 + 6? 8 + 37 + 6? 8 + 67 + 6?
16. 9 + 9? 8 + 4? 9 + 9 + 4? 19 + 9 + 4?
17. 29 + 9 + 4? 79 + 9 + 4? 49 + 9 + 4?
18. 9 + 49 + 4? 9 + 69 + 4? 9 + 59 + 4?
19. 5 + 8? 3 + 9? 5 + 8 + 9? 15 + 8 + 9?
20. 5 + 18 + 9? 5 + 68 + 9? 5 + 38 + 9?
21. 5 + 48 + 9? 75 + 8 + 9? 55 + 8 + 9?
22. 6 + 7? 3 + 6? 9 + 5? 6 + 7 + 6 + 5?
23. 16 + 7 + 6 + 5? 46 + 7 + 6 + 5?
24. 6 + 47 + 6 + 5? 6 + 27 + 6 + 5?
25. 6 + 57 + 6 + 5? 66 + 7 + 6 + 5?
26. 3 + 8? 1 + 6? 7 + 7? 3 + 8 + 6 + 7?
27. 13 + 8 + 6 + 7? 43 + 8 + 6 + 7?
28. 73 + 8 + 6 + 7? 23 + 8 + 6 + 7?
29. 3 + 28 + 6 + 7? 3 + 58 + 6 + 7?
30. 9 + 5? 4 + 6? 4 + 7? 9 + 5 + 6 + 4 + 7?
31. 49 + 5 + 6 + 4 + 7? 39 + 5 + 6 + 4 + 7?
32. 69 + 5 + 6 + 4 + 7? 9 + 65 + 6 + 4 + 7?
33. 9 + 25 + 6 + 4 + 7? 9 + 35 + 6 + 4 + 7?
34. 8 + 8? 6 + 7? 3 + 9? 8 + 8 + 7 + 9?
35. 18 + 8 + 7 + 9? 58 + 8 + 7 + 9?
36. 28 + 8 + 7 + 9? 48 + 8 + 7 + 9?
37. 68 + 8 + 7 + 9? 38 + 8 + 7 + 9?
38. 7 + 5? 2 + 9? 1 + 7? 8 + 8? 7 + 5 + 9 + 7 + 8?
39. 17 + 5 + 9 + 7 + 8 + 1 + 5 + 9 + 7 + 8 + 1 + 5 + 9 + 7?

**C.** 1. 7 + 8? 5 + 8? 7 + 8 + 8 + 4 + 8 + 8 + 4 + 8 + 8 + 7 + 7 + 8 + 8?

2. 5 + 9? 15 + 9 + 1 + 9 + 6 + 5 + 9 + 6 + 8 + 2 + 5 + 9 + 1 + 9?

3. 7 + 6? 6 + 7? 7 + 6 + 3 + 7 + 4 + 6 + 3 + 7 + 4 + 6 + 3 + 7 + 3 + 7 + 4 + 6?

4. 8 + 3 + 7 + 3 + 2 + 8 + 7 + 3 + 7 + 3 + 2 + 8 + 2 + 8?

5. 5 + 9 + 8 + 7 + 6 + 9 + 8 + 7 + 6 + 9 + 8 + 7 + 6?

6. 8 + 6 + 7 + 5 + 9 + 8 + 8 + 7 + 6 + 7 + 5 + 9 + 8?

7. 9 + 8 + 7 + 3 + 4 + 5 + 4 + 7 + 2 + 8 + 7 + 5 + 4 + 6 + 8 + 7?

8. 4 + 5 + 6 + 7 + 6 + 3 + 8 + 6 + 7 + 7 + 3 + 2 + 5 + 7 + 6?

9. 7 + 4 + 5 + 9 + 2 + 4 + 7 + 3 + 2 + 5 + 6 + 1 + 2 + 8 + 5 + 7?

10. 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9?

11. 7 + 5 + 4 + 8 + 3 + 2 + 4 + 5 + 9 + 3 + 2 + 7 + 7 + 5?

**D.** 1. Add as many twos as you can to 2 without obtaining a greater sum than 50. *Solution.* 2 + 2 = 4, + 2 = 6, + 2 = 8, &c.

2. Add twos to 1 in the same manner.

3. Add as many threes as you can to 3 without obtaining a greater sum than 60.

4. Add threes to 2 in the same manner; to 1.

5. Add as many fours as you can to 4 without obtaining a greater sum than 80.

6. In the same manner add fours to 3; to 1; to 2.

7. Add as many fives as you can to 5 without obtaining a greater sum than 50.

8. In the same manner add fives to 4; to 2; to 3; to 1.

9. Add as many sixes as you can to 6 without obtaining a greater sum than 100.

10. In the same manner add sixes to 1; to 4; to 3; to 5; to 2.

11. Add as many sevens as you can to 7 without obtaining a greater sum than 100.

12. In the same manner add sevens to 3; to 1; to 6; to 4; to 5.

13. Add as many eights as you can to 8 without obtaining a greater sum than 100.

14. In the same manner add eights to 7; to 1; to 5; to 3; to 2; to 4; to 6.

15. Add as many nines as you can to 9 without obtaining a greater sum than 100.

16. In the same manner add nines to 5; to 7; to 3; to 1; to 4; to 8; to 6; to 2.

17. Add as many tens as you can to 10 without obtaining a greater sum than 100.

18. In the same manner add tens to 3; to 5; to 8; to 1; to 7; to 2; to 9; to 4; to 6.

NOTE. The pupil should now be taught to count by twos, threes, &c., beginning as in above examples. Thus, 2, 4, 6, 8, 10, &c.; 1, 3, 5, 7, 9, &c.; 3, 6, 9, 12, 15, &c.; 2, 5, 8, 11, 14, &c. This is a *very valuable* exercise, and should be continued till all the combinations are perfectly familiar. Facility in addition requires that the results only should be given.

**E.** 1. $15-8+40$? $40+15-8$? $55-8$?
2. $15-8+70$? $70+15-8$? $85-8$?
3. $15-8+20$? $20+15-8$? $35-8$?
4. $25-8$? $75-8$? $95-8$? $65-8$?
5. $13-4+20$? $20+13-4$? $33-4$? $53-4$?
6. $73-4$? $43-4$? $93-4$? $23-4$? $83-4$?
7. $11-7+40$? $40+11-7$? $51-7$? $21-7$?
8. $91-7$? $61-7$? $31-7$? $71-7$? $81-7$?
9. $12-6+80$? $80+12-6$? $92-6$? $22-6$?
10. $52-6$? $82-6$? $72-6$? $32-6$? $42-6$?
11. $18-9+50$? $50+18-9$? $68-9$? $98-9$?
12. $48-9$? $28-9$? $88-9$? $38-9$? $78-9$?
13. $16-7+30$? $30+16-7$? $46-7$? $26-7$?
14. $86-7$? $96-7$? $36-7$? $76-7$? $56-7$?
15. $97-8$? *Solution.* $97-8=80+17-8$, and $17-8=9$, which added to $80=89$, Ans.
16. $56-7$? $37-9$? $54-6$? $42-5$? $87-8$?
17. $91-5$? $83-6$? $52-7$? $23-8$? $45-7$?
18. $64-8$? $27-8$? $53-7$? $24-7$? $94-7$?
19. $61-8$? $55-9$? $76-8$? $94-9$? $85-7$?

**To the Teacher.**—After the solution given above is well understood, let the questions be solved as follows: [Ex. 15.] 17 — 8 = 9. Therefore, 97 — 8 = 89. The object of all the above forms, as also of those under the following letter, is to lead the pupil, in subtracting, to refer to the simplest combinations, or, in other words, to apply the decimal system of numbers.

**F.** 1. 14 — 8? 16 — 7? 44 — 8 — 7? *Solution.* 44 — 8 = 36, — 7 = 29, Ans.

2. 54 — 8 — 7? 94 — 8 — 7? 34 — 8 — 7?
3. 84 — 8 — 7? 64 — 8 — 7? 24 — 8 — 7?
4. 13 — 5? 18 — 9? 23 — 5 — 9? 63 — 5 — 9?
5. 83 — 5 — 9? 43 — 5 — 9? 93 — 5 — 9?
6. 73 — 5 — 9? 33 — 5 — 9? 53 — 5 — 9?
7. 17 — 9? 18 — 9? 27 — 9 — 9? 97 — 9 — 9?
8. 67 — 9 — 9? 37 — 9 — 9? 47 — 9 — 9?
9. 77 — 9 — 9? 57 — 9 — 9? 87 — 9 — 9?
10. 12 — 7? 15 — 6? 22 — 7 — 6? 82 — 7 — 6?
11. 92 — 7 — 6? 32 — 7 — 6? 42 — 7 — 6?
12. 72 — 7 — 6? 52 — 7 — 6? 62 — 7 — 6?
13. 16 — 8? 18 — 9? 46 — 8 — 9? 96 — 8 — 9?
14. 86 — 8 — 9? 26 — 8 — 9? 56 — 8 — 9?
15. 36 — 8 — 9? 76 — 8 — 9? 66 — 8 — 9?
16. 14 — 7? 17 — 8? 34 — 7 — 8? 84 — 7 — 8?
17. 24 — 7 — 8? 44 — 7 — 8? 94 — 7 — 8?
18. 64 — 7 — 8? 54 — 7 — 8? 74 — 7 — 8?
19. 11 — 7? 14 — 5? 91 — 7 — 5 — 8 — 7 — 5 — 8 — 7 — 5 — 8 — 7 — 5 — 8 — 7?
20. 81 — 7 — 5 — 8 — 7 — 5 — 8 — 7 — 5 — 8 — 7 — 5 — 8?
21. 13 — 7? 16 — 9? 93 — 7 — 9 — 4 — 7 — 9 — 4 — 7 — 9 — 4 — 7 — 9 — 4 — 7?
22. 83 — 7 — 9 — 4 — 7 — 9 — 4 — 7 — 9 — 4 — 7 — 9 — 4?
23. 12 — 8? 14 — 6? 92 — 8 — 6 — 6 — 8 — 6 — 6 — 8 — 6 — 6 — 8 — 6 — 6 — 8?
24. 82 — 8 — 6 — 6 — 8 — 6 — 6 — 8 — 6 — 6 — 8 — 6 — 6?

25. 15 — 9? 16 — 8? 95 — 9 — 8 — 3 — 9 — 8 — 3 — 9 — 8 — 3 — 9 — 8 — 3 — 9?

26. 85 — 9 — 8 — 3 — 9 — 8 — 3 — 9 — 8 — 3 — 9 — 8 — 3?

27. 17 — 9? 18 — 9? 87 — 9 — 9 — 2 — 9 — 9 — 2 — 9 — 9 — 2 — 9 — 9 — 2?

28. 97 — 9 — 9 — 2 — 9 — 9 — 2 — 9 — 9 — 2 — 9 — 9 — 2 — 9?

**G.** 1. 9 + 8 + 6 + 7 + 5 + 9 + 8 + 9 + 5 + 7 — 7 — 5 — 9 — 8 — 9 — 5 — 7 — 6 — 8?

2. 8 + 6 + 5 + 7 + 9 + 8 + 3 + 5 + 4 + 6 + 8 + 3 — 3 — 8 — 6 — 4 — 5 — 3 — 8 — 9 — 7 — 5 — 6?

3. 17 + 5 + 8 + 6 + 3 + 9 + 7 + 8 + 9 + 6 + 4 — 4 — 6 — 9 — 8 — 7 — 9 — 3 — 6 — 8 — 5 — 6?

4. 23 + 4 + 6 + 8 + 2 + 7 + 5 + 3 + 9 + 4 + 6 + 8 + 9 — 9 — 8 — 6 — 4 — 9 — 3 — 5 — 7 — 2 — 8 — 6 — 4 — 7 — 5?

5. 20 + 7 + 4 + 8 + 6 + 5 + 3 + 9 + 8 + 4 + 7 + 9 + 2 + 7 — 7 — 2 — 9 — 7 — 4 — 8 — 9 — 3 — 5 — 6 — 8 — 4 — 7 — 9 — 4?

6. 15 + 9 + 4 + 4 + 8 + 6 + 8 + 9 + 5 + 5 + 8 + 8 + 9 — 9 — 8 — 9 — 5 — 5 — 9 — 8 — 6 — 8 — 4 — 4 — 9 — 6?

7. 38 + 9 + 7 — 6 — 5 — 9 — 3 + 8 + 6 + 9 — 5 — 8 + 6 — 9 + 5 — 4 + 9 — 3 — 8 — 5?

8. 54 — 9 — 6 — 7 — 8 + 3 + 3 + 5 + 9 + 9 + 6 — 8 — 7 + 8 — 6 — 5 — 4?

9. 67 — 8 — 5 — 8 — 6 — 3 + 5 + 9 — 8 — 7 + 6 + 3 — 9 — 3 — 9 — 5 — 2 — 4 — 3?

10. 71 — 6 — 9 — 3 — 8 — 4 — 2 + 5 + 7 + 3 — 8 — 5 — 6 — 4 + 7 — 3 — 9?

**H.** 1. Subtract as many twos as you can from 50. Solution: 50 — 2 = 48, — 2 = 46, — 2 = 44, &c.

2. Subtract as many twos as you can from 49.

3. Subtract as many threes as you can from 60; from 62; from 58.

4. Subtract as many fours as you can from 80; from 78; from 81; from 79.

5. Subtract as many fives as you can from 50; from 46; from 54; from 48.

6. Subtract as many sixes as you can from 100; from 97; from 98; from 95; from 99; from 96.

7. Subtract as many sevens as you can from 100; from 95; from 98; from 94; from 99; from 97; from 96.

8. Subtract as many eights as you can from 100; from 96; from 98; from 94; from 97; from 95; from 93; from 99.

9. Subtract as many nines as you can from 100; from 92; from 99; from 94; from 97; from 95; from 93; from 98; from 96.

10. Subtract as many tens as you can from 100; from 95; from 98; from 91; from 93; from 97; from 92; from 96; from 94; from 99.

NOTE. — The above examples should be performed by the pupil, after he has become familiar with the process, as were those in letter D. of this section, the results only being given: thus, to subtract twos from 50, let him say, 50, 48, 46, 44, 42, &c.

Add the following columns.

**I.**

| First. | Second. | Third. | Fourth. | Fifth. | Sixth. | Seventh. | Eighth. | Ninth. | Tenth. |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 9 | 3 | 5 | 4 | 9 | 4 | 9 | 3 | 6 |
| 8 | 3 | 9 | 8 | 6 | 7 | 8 | 8 | 4 | 8 |
| 7 | 9 | 5 | 7 | 4 | 1 | 6 | 5 | 8 | 5 |
| 9 | 8 | 7 | 6 | 5 | 3 | 5 | 1 | 6 | 4 |
| 3 | 5 | 7 | 5 | 7 | 5 | 7 | 2 | 5 | 7 |
| 4 | 8 | 8 | 4 | 9 | 8 | 9 | 4 | 3 | 9 |
| 8 | 7 | 8 | 3 | 8 | 4 | 8 | 7 | 2 | 5 |
| 5 | 3 | 6 | 2 | 6 | 2 | 3 | 6 | 9 | 3 |
| 9 | 3 | 6 | 1 | 2 | 6 | 2 | 5 | 7 | 4 |
| 8 | 6 | 5 | 9 | 5 | 9 | 6 | 2 | 8 | 6 |
| 4 | 7 | 5 | 8 | 9 | 6 | 1 | 8 | 4 | 9 |
| 8 | 5 | 9 | 7 | 4 | 3 | 5 | 6 | 8 | 5 |
| 9 | 2 | 9 | 6 | 3 | 7 | 7 | 7 | 1 | 8 |

**J.** 1. A milkman left one morning, 2 quarts of milk at one place, 8 quarts at another, 3 at a third, 4 at a fourth, 2 at a fifth, 3 at a sixth, 5 at a seventh, 3 at an eighth, 1 at a ninth, 4 at a tenth, 7 at an eleventh, 6 at a twelfth, 1 at a thirteenth, 10 at a fourteenth,and 2 at a fifteenth. How many quarts did he leave?

2. From Boston to Dorchester is 3 miles, from Dorchester to Quincy 5 miles, from Quincy to Braintree 3 miles, from Braintree to South Weymouth 3 miles, from South Weymouth to Abington 4, from Abington to Hanson 6, from Hanson to Halifax 4, from Halifax to Kingston 5, from Kingston to Plymouth 4. Required the distance from Boston to Plymouth?

3. A shoemaker bought a dozen kid-skins, from which he cut uppers for ladies' shoes; from the first, he cut 4 pairs, from the second 6, from the third 3, from the fourth 8, from the fifth 5, from the sixth 7, from the seventh 10, from the eighth 5, from the ninth 8, from the tenth 7, from the eleventh 9, and from the twelfth 4. How many did he cut from all? He had 8 calf-skins, from 4 of which he cut boots, and from the remainder, boys' shoes; from one calf-skin he cut 2 pairs of boots, from another 3, from another 5, and from another 4; from a fifth he cut 6 pairs of boys' shoes, from a sixth 8 pairs, from a seventh 9 pairs, and from the other 9 pairs. How many pairs of boots did he cut? of shoes?

4. Mr. Dean made 8 pairs of ladies' shoes on Monday, 6 pairs on Tuesday, 10 pairs on Wednesday, 7 pairs on Thursday, 9 pairs on Friday, and 8 pairs on Saturday. How many did he make during the week? Mr. Knight made on Monday 9 pairs, on Tuesday 7 pairs, on Wednesday 4 pairs, on Thursday 10 pairs, on Friday 5 pairs, and on Saturday 8. How many pairs did he make during the week? How many less than Mr. Dean?

5. Henry had 96 cents; he bought some ink for 8 cents, some paper for 9 cents, some wafers for 6 cents, some pens for 7 cents, a pen-holder for 9 cents, a drawing-pencil for 8 cents, a writing-book for 7 cents, and a drawing-book for 20 cents. How much did he have left after paying for the ink? for the paper? wafers? pens? pen-holder? drawing-pencil? writing-book? drawing-book?

6. John Smith, Jr., of Arkansas, bought 50 bushels of cranberries. He sold at one time 6 bushels for 9 dollars, at another 5 bushels for 8 dollars, at another 2 bushels for 3 dollars, at another 7 bushels for 10 dollars, at another 20 bushels for 30 dollars, and at another 5 bushels for 7 dollars, keeping the rest for his own use. How many did he have left after selling each lot? How many did he keep for his own use? How many dollars did he receive for those he sold?

7. Martha and Mary had a beautiful flower-garden in which they had the following plants: Martha had 9 Dahlias, 8 Tulips, 7 Geraniums and 6 Verbenas. Mary had 5 Columbines, 4 Cactuses, 8 Hyacinths. How many plants had Martha? How many had Mary? Martha gave to Mary 5 Dahlias, 4 Tulips, 3 Geraniums and 2 Verbenas. How many plants had Martha after each gift? How many had Mary? Mary then gave Martha 3 Columbines, 2 Cactuses and 4 Hyacinths. How many had each then?

8. Arthur had 75 cents; he found 7 cents, gave 5 cents to a poor boy, earned 8 cents picking berries, and his mother gave him 6 cents. He then paid 7 cents for a writing-book, his father gave him 8 cents, he gave 3 cents for a pencil, earned 8 cents by doing errands, gave 6 cents for a picture, received 5 cents for picking a quart of cherries, and gave 9 cents to a poor man who had broken his arm. How many cents had he then? How many did he spend for himself? How many did ~~he~~ give away? How many did he earn?

9. A news-boy bought on Monday morning, 8 copies of the Boston Post, 9 of the Atlas, 3 of the Herald, 10 of the Bee, 10 of the Daily Mail, 9 of the Times, 10 of the Commonwealth, 11 of the Transcript, 8 of the Advertiser, 10 of the Traveller, 8 of the Journal, and 7 of the Courier. How many papers did he buy in all? He sold all the copies of the Post, Atlas, Commonwealth, Transcript, and Traveller that he had, and he sold 1 copy of the Herald, 8 of the Bee, 9 of the Mail, 7 of the Times, 5 of the Advertiser, 6 of the Journal and 4 of the Courier. How many papers did he sell? How many remained unsold?

10. Last month I had 27 dollars. I have since received 9 dollars for flour, 8 dollars for sugar, 7 dollars for raisins, paid 6 dollars for labor, 4 dollars for a pair of boots, 4 dollars for a hat, 6 dollars for a vest, 8 dollars for a pair of pantaloons, received 5 dollars for meal, 8 dollars for corn, 7 dollars for oats, paid 8 dollars for railroad fare, 7 dollars for horse-hire, 9 dollars for other travelling expenses, received 8 dollars for apples, 9 for potatoes, 3 for molasses, 2 for figs, 1 for spices, 8 for tea, 6 for coffee, 5 for butter, and I have given away to poor persons 6 dollars. How many dollars have I now?

11. The distances from each other of the several stations on the Boston and Providence Railroad are as follows, viz.: — Boston to Roxbury 2 miles, Roxbury to Readville 5 miles, Readville to Canton 6 miles, Canton to Sharon 4 miles, Sharon to Foxborough 4 miles, Foxborough to Mansfield 3 miles, Mansfield to Attleborough 7 miles, Attleborough to Pawtucket 8 miles,

Pawtucket to Providence 4 miles. How far are each of the above stations from Boston? from Providence? from Attleborough? from Canton?

12. On the Eastern Railroad it is 4 miles from Boston to N. Chelsea, 5 miles from N. Chelsea to Lynn, 5 from Lynn to Salem, 2 from Salem to Beverly, 4 from Beverly to Wenham, 5 from Wenham to Ipswich, 4 from Ipswich to Rowley, 5 from Rowley to Newburyport, 2 from Newburyport to Salisbury, 4 from Salisbury to Seabrook, 2 from Seabrook to Hampton Falls, 4 from Hampton Falls to North Hampton, 3 from North Hampton to Greenfield, and 5 from Greenfield to Portsmouth. How far is each of the above stations from Boston? from Portsmouth? from Salem? from Newburyport?

13. Three sisters, named Ellen, Laura, and Eliza, went out to search for blackberries at a time when they were very scarce. Ellen found 23 in one place, 8 in another, and 5 in another, she then ate 7, and gave 9 to a little boy. How many did she have left? She afterwards found 6 at one time, 7 at another, and 5 at another, and having the ill luck to fall she lost 9. How many did she have left? Of these she gave 9 to her father, 9 to her mother, 7 to her aunt, and the rest to her uncle. How many did she give to her uncle? Laura found 45 at one time, 7 at another, 6 at another, and 5 at another, when feeling tired she sat down to rest. How many did she have when she sat down to rest? She ate 4, put 8 into a hole in a tree for the squirrels to eat, and threw 5 to some birds. How many did she have left? On her way home she found 7, but meeting her brothers Jarvis and Arthur, and her cousins Robert and Joseph, she gave Jarvis 7, Arthur 8, Robert 9, and the rest to Joseph. How many did she give to Joseph. Eliza found 36 berries at one time, 5 at another, and 8 at another, when meeting Hannah and Martha, she gave Hannah 9, and Martha 8. How many did she have left? She then found 5, had 7 given her by Charles, 6 by Samuel, and 10 by Harry. How many did she have at this time? She gave 9 of her berries to Miss Price, her former teacher, 8 to Miss Metcalf, who was then her teacher, and sent the rest to a sick school-mate. How many did she send to her school-mate?

14. The next day they went out again, intending to try and see who would get the most. Ellen picked 8 blackberries in one place, 33 in another, 4 in another, 7 in another, and 6 in another. How many did she pick? Laura first picked 9, then 7, then 6, then 4, then 8, then 3, then 9, then 6, and then 6. How many did she pick? Eliza first picked 3, then 4, then 4,

then 6, then 8, then 6, then 8, then 7, then 8, and then 4. How many did she pick? Which picked the most, Ellen, Laura, or Eliza? Ellen gave 8 berries to her father, and the rest to her mother. Laura gave 9 of hers to her cousin Joseph, 9 to Robert, 9 to Mr. Pond, 9 to Mr. Keith, 9 to Mr. Lewis, and kept the rest. Eliza gave 10 to Miss Price, 10 to Miss Metcalf, and the rest to her father. How many did Ellen give to her mother? How many did Laura keep? How many did Eliza give to her father?

15. Jarvis, Arthur, Robert, and Joseph went on a fishing excursion; Jarvis caught 8 pickerel, 9 perch, 7 trout, 8 shiners, 9 roach, 7 pouts, and 3 eels. How many fish did he catch? The pickerel weighed 12 pounds, the perch 6, the trout 5, the shiners 2, the roach 3, the pouts 2, and the eels 2. How many pounds did they all weigh? Arthur caught 7 pickerel, 10 perch, 9 trout, 9 shiners, 7 roach, 8 pouts, and 1 eel. How many fish did he catch? His pickerel weighed 7 pounds, his trout 7, his shiners 2, his roach 2, his pouts 4, and his eel 2. How many pounds did Arthur's fish weigh? Robert caught 4 pickerel, 3 trout, 8 shiners, 9 roach, 10 pouts, and 6 eels. How many fish did he catch? His pickerel weighed 3 pounds, his trout 4 pounds, his shiners 2 pounds, his roach 3 pounds, his pouts 9 pounds, and his eels 6 pounds. How much did Robert's fish weigh? Joseph caught only 27 shiners, 7 roach, and 10 very large pouts. How many fish did he catch? His fish weighed 19 pounds, of which the shiners weighed 3 pounds, the roach 2 pounds, and the pouts the rest. How much did the pouts weigh? How many pounds of pickerel did all catch? of perch? of trout? of shiners? of roach? of pouts? of eels?

16. While on their fishing excursion, Jarvis gathered 8 pond lilies in one place, 10 in another, 4 in another, and 3 in another. Robert gathered 6 in one place, 5 in another, 8 in another, and 7 in another. Joseph gathered 7 in one place, 6 in another, 9 in another, and 4 in another. Arthur gathered 6 in one place, 10 in another, 4 in another, and 3 in another. How many lilies did all gather? They put them all together, agreeing to let each one use or give away as many as he chose till all should be gone. Jarvis gave his father 5, his mother 7, and his cousin Myra 10. How many were there left? Arthur gave his father 9, his mother 9, and his sister Eliza 7. How many were there left? Robert gave Ellen 8, Laura 10, and Eliza 5. How many were then left? Joseph gave Ellen 9, Laura 7, and Eliza 8, after which he gave what there were left to Laura. How many did he give to Laura the second time?

## SECTION IV.

**A.** 1. 10 tens + 10 tens, or 20 tens = 100 + 100 = 2 hundred, called TWO HUNDRED, written 200.

2. 20 tens + 10 tens, or 30 tens = 200 + 100 = 3 hundred, called THREE HUNDRED, written 300.

3. 30 tens + 10 tens, or 40 tens = 300 + 100 = 4 hundred, called FOUR HUNDRED, written 400.

4. 40 tens + 10 tens, or 50 tens = 400 + 100 = 5 hundred, called FIVE HUNDRED, written 500.

5. 50 tens + 10 tens, or 60 tens = 500 + 100 = 6 hundred, called SIX HUNDRED, written 600.

6. 60 tens + 10 tens, or 70 tens = 600 + 100 = 7 hundred, called SEVEN HUNDRED, written 700.

7. 70 tens + 10 tens, or 80 tens = 700 + 100 = 8 hundred, called EIGHT HUNDRED, written 800.

8. 80 tens + 10 tens, or 90 tens = 800 + 100 = 9 hundred, called NINE HUNDRED, written 900.

9. 90 tens + 10 tens, or 100 tens = 900 + 100 = 10 hundred, called ONE THOUSAND, written 1000.

NOTE. The third place towards the left from the decimal point is called the place of hundreds, and the fourth place that of thousands, because the figures in these places represent hundreds and thousands respectively. Thus, 1 in the third place represents 1 hundred, 5 in the third place represents 5 hundred, and 7, 7 hundred; but when in the fourth place, 1 represents 1 thousand, 5, 5 thousand, 7, 7 thousand, &c.

**B.** 1. What do we call 20 tens? 50 tens? 100 tens? 80 tens? 30 tens? 60 tens? 40 tens? 70 tens? 90 tens?

2. How is two hundred written by figures? Five hundred? Nine hundred? Ten hundred or one thousand? Three hundred? Seven hundred? Four hundred? Six hundred? Eight hundred?

3. How many tens in 500? in 900? in 700? in 200? in 800? in 400? in 100? in 600? in 300?

**C.** 1. 10 tens + 1 ten, or 11 tens = 1 hundred, and 1 ten, called ONE HUNDRED AND TEN, written 110.

2. 10 tens + 2 tens, or 12 tens = 1 hundred, and 2 tens, called ONE HUNDRED AND TWENTY, written 120.

3. 10 tens + 3 tens, or 13 tens = 1 hundred, and 3 tens, called ONE HUNDRED AND THIRTY, written 130.

4. 10 tens + 4 tens, or 14 tens = 1 hundred, and 4 tens, called ONE HUNDRED AND FORTY, written 140.

5. 10 tens + 5 tens, or 15 tens = 1 hundred, and 5 tens, called ONE HUNDRED AND FIFTY, written 150.

6. 10 tens + 6 tens, or 16 tens = 1 hundred, and 6 tens, called ONE HUNDRED AND SIXTY, written 160.

7. 10 tens + 7 tens, or 17 tens = 1 hundred, and 7 tens, called ONE HUNDRED AND SEVENTY, written 170.

8. 10 tens + 8 tens, or 18 tens = 1 hundred, and 8 tens, called ONE HUNDRED AND EIGHTY, written 180.

9. 10 tens + 9 tens, or 19 tens = 1 hundred, and 9 tens, called ONE HUNDRED AND NINETY, written 190.

10. 20 tens + 1 ten, or 21 tens = 2 hundred, and 1 ten, called TWO HUNDRED AND TEN, written 210.

11. 20 tens + 3 tens, or 23 tens = 2 hundred, and 3 tens, called TWO HUNDRED AND THIRTY, written 230.

12. 30 tens + 5 tens, or 35 tens = 3 hundred, and 5 tens, called THREE HUNDRED AND FIFTY, written 350.

13. 50 tens + 7 tens, or 57 tens = 5 hundred, and 7 tens, called FIVE HUNDRED AND SEVENTY, written 570.

14. 60 tens + 3 tens, or 63 tens = 6 hundred, and 3 tens, called SIX HUNDRED AND THIRTY, written 630.

15. 70 tens + 30 tens, or 100 tens = 7 hundred, and 3 hundred, called ONE THOUSAND, written 1000.

**D.** 1. What do we call 11 tens? 17 tens? 19 tens? 12 tens? 23 tens? 57 tens? 63 tens? 81 tens? 99 tens? 45 tens? 31 tens? 87 tens? 93 tens? 52 tens? 79 tens? 26 tens? 14 tens? 64 tens? 18 tens? 43 tens?

2. How is one hundred and ten written by figures?

3. One hundred and fifty? Three hundred and twenty?

4. Five hundred and seventy? Eight hundred and sixty?

5. Nine hundred and ten? Seven hundred and forty?
6. Nine hundred and eighty? Three hundred and ninety?
7. Four hundred and thirty? Two hundred and fifty?
8. Six hundred and ten? One hundred and twenty?
9. Seven hundred and sixty? Six hundred and ninety?
10. Three hundred and seventy?
11. How many tens in 110? in 420? in 680? in 530? in 990? in 310? in 1000? in 370? in 210? in 570? in 490? in 940? in 720? in 270? in 560? in 650? in 910? in 190?

**E.** 1. 9 tens + 8 tens = how many tens? Called what? How many then are 90 + 80?

2. 7 tens + 6 tens = how many tens? Called what? How many then are 70 + 60?

3. 26 tens + 8 tens = how many tens? Called what? How many then are 260 + 80?

Read the following, supplying appropriate numbers in place of the stars.

4. 37 tens + 9 tens = * tens, or *; then 370 + 90 = *.
5. 81 tens + 5 tens = * tens, or *; then 810 + 50 = *.
6. 82 tens + 2 tens = * tens, or *; then 820 + 20 = *.
7. 27 tens + 6 tens = * tens, or *; then 270 + 60 = *.
8. 45 tens + 8 tens = * tens, or *; then 450 + 80 = *.
9. 74 tens + 6 tens = * tens, or *; then 740 + 60 = *.
10. 22 tens + 9 tens = * tens, or *; then 220 + 90 = *.
11. 17 tens + 5 tens = * tens, or *; then 170 + 50 = *.
12. 53 tens + 6 tens = * tens, or *; then 530 + 60 = *

**F.** 1. 17 tens — 9 tens = how many tens? Called what? How many then are 170 — 90?

2. 32 tens — 5 tens = how many tens? Called what? How many then are 320 — 50?

3. 59 tens — 7 tens = how many tens? Called what? How many then are 590 — 70?

Read the following, supplying appropriate numbers in place of the stars.

4. 40 tens — 6 tens = 34 tens, or 340; then 400 — 60 = 340.
5. 94 tens — 7 tens = * tens, or *; then 940 — 70 = *.
6. 31 tens — 8 tens = * tens, or *; then 310 — 80 = *.
7. 60 tens — 3 tens = * tens, or *; then 600 — 30 = *.
8. 50 tens — 8 tens = * tens, or *; then 500 — 80 = *.
9. 81 tens — 3 tens = * tens, or *; then 810 — 30 = *.
10. 48 tens — 6 tens = * tens, or *; then 480 — 60 = *.
11. 12 tens — 4 tens = * tens, or *; then 120 — 40 = *.
12. 63 tens — 5 tens = * tens, or *; then 630 — 50 = *.

**G.** 1. 9 tens + 7 tens + 8 tens + 9 tens — 4 tens — 6 tens? 90 + 70 + 80 + 90 — 40 — 60?
2. 18 tens + 6 tens + 9 tens + 9 tens + 8 tens — 7 tens — 9 tens — 6 tens? 180 + 60 + 90 + 90 + 80 — 70 — 90 — 60?
3. 36 tens — 9 tens — 7 tens + 4 tens + 8 tens + 3 tens — 9 tens? 360 — 90 — 70 + 40 + 80 + 30 — 90?
4. 450 + 60 + 70 — 30 — 90 + 70 — 60 — 50?
5. 50 + 80 + 70 + 50 — 80 + 90 + 70 + 50?
6. 20 + 90 + 50 — 80 + 70 — 90 + 80 + 90?
7. 70 + 80 + 40 — 30 + 70 + 60 — 90 — 20?
8. 710 — 80 — 50 + 80 + 70 — 90 — 20 + 80?
9. 220 + 90 + 20 — 70 + 50 — 80 + 20 + 90?
10. 900 — 50 — 60 + 30 — 20 — 70 + 90 — 80?
11. 700 + 60 + 90 — 80 — 90 + 30 + 50 — 70?

**H.** 1. Add as many twenties as you can to 20 without obtaining a greater sum than 200. *Solution.* 20 + 20 = 40, + 20 = 60, + 20 = 80, &c.
2. In the same manner add twenties to ten.
3. Add as many thirties as you can to 30 without obtaining a greater sum than 300.
4. In the same manner add thirties to 10; to 20.
5. Add as many forties as you can to 40 without obtaining a greater sum than 400.
6. In the same manner add forties to 10; to 30; to 20.
7. Add as many fifties as you can to 50 without obtaining a greater sum than 500.

8. In the same manner add fifties to 10; to 30; to 20; to 40.

9. Add as many sixties as you can to 60 without obtaining a greater sum than 600.

10. In the same manner add sixties to 40; to 10; to 50; to 20; to 30.

11. Add as many seventies as you can to 70 without obtaining a greater sum than 700.

12. In the same manner add seventies to 50; to 10; to 60; to 20; to 40; to 30.

13. Add as many eighties as you can to 80 without obtaining a greater sum than 800.

14. In the same manner add eighties to 50; to 30; to 70; to 10; to 60; to 20; to 40.

15. Add as many nineties as you can to 90 without obtaining a greater sum than 900.

16. In the same manner add nineties to 80; to 10; to 60; to 30; to 40; to 50; to 20; to 70.

17. *Count* by twenties, thirties, &c., beginning as in the above examples. Thus, 20, 40, 60, 80, 100, &c.

**I.** 1. Subtract as many twenties as you can from 200. *Solution.* 200 — 20 = 180, — 20 = 160, — 20 = 140, &c., &c.

2. Subtract as many twenties as you can from 190.

3. Subtract as many thirties as you can from 300; from 180; from 190.

4. Subtract as many forties as you can from 400; from 370; from 390; from 380.

5. Subtract as many fifties as you can from 500; from 470; from 490; from 460; from 480.

6. Subtract as many sixties as you can from 600; from 550; from 580; from 590; from 570; from 560.

7. Subtract as many seventies as you can from 700; from 650; from 670; from 690; from 680; from 660; from 640.

8. Subtract as many eighties as you can from 800; from 770; from 740; from 730; from 760; from 790; from 780; from 750.

9. Subtract as many nineties as you can from 900; from 820; from 850; from 880; from 890; from 860; from 830; from 840; from 870.

Perform the above, repeating only the results. Thus, in the first example, 200, 180, 160, 140, &c.

**II.** 1. In Mr. Chaplin's library there are 80 volumes of History, 70 of Biography, 90 of Poetry, 60 of Essays, 30 of Travels, 80 on Science, 90 on Theology, and 90 on Miscellaneous subjects. How many books are there in his library?

2. Mr. Jackson had 430 dollars; he paid 100 dollars for a horse, 80 for a wagon, and 30 for a harness. How many dollars had he left?

3. Alfred found 810 nuts; he ate 40 and gave away 30. How many had he left? He afterwards sold 90. How many did he then have left? The rats carried away 60 of them? How many had he then? He gave 80 to his little sister Agnes. How many had he then?

4. In a work on Natural Philosophy, 100 pages were devoted to Mechanics, 60 to Hydrostatics, 80 to Pneumatics, 10 to Acoustics, 40 to Optics, 40 to Electricity, 30 to Magnetism, and 10 to preface and title pages. How many pages were there in the work?

5. In an Arithmetic, 20 pages were devoted to Addition and Subtraction, 20 to Multiplication and Division, 20 to Compound Numbers, 30 to Fractions, 10 to Interest and Discount, 10 to Proportion, 10 to Compound Proportion, 10 to Alligation, 20 to Evolution, 10 to Arithmetical Progression, 10 to Geometrical Progression, and 50 to Miscellaneous Subjects. How many pages did the work contain?

6. In a certain library there was a book-case containing 4 shelves, on the first of which, there were 30 volumes, on the second 40, on the third 70, and on the fourth 90. How many books were there in the case? How many more on the fourth shelf than on the third? Than on the second? Than on the first? How many more on the third shelf than on the second? Than on the first? How many more on the second shelf than on the first?

7. A gentleman having 2 lots of land, expended on each lot, labor and money to the amount of $20. On one lot he raised 200 bushels of potatoes which he sold for 100 dollars. On the other he raised 30 bushels of corn which he sold for 1 dollar per bushel, 250 pumpkins which he sold for 10 dollars, 60 bushels of onions which he sold for 1 dollar per bushel, and 60 bushels of beets and carrots which he sold for 20 dollars. How much money did he receive for his produce? What did he gain on the two lots of land? How much more did he receive for the produce of one lot than for that of the other?

8. Mr. Hovey owned 520 acres of land, but he sold 60 acres, and gave his son 80 acres. How many acres had he left?

Only 90 acres of his remaining land are under cultivation. How many acres are not under cultivation? 90 acres of his uncultivated land are low and swampy, and the remainder is upland. How many acres of it are upland?

9. Mr. J. M. Baker contributed 200 dollars towards repairing a church, Mr. Noyes contributed 100, Mr. Thompson 50, Mr. Tubbs 50, Mr. Fisher 60, Mr. Morse 80, Mr. Draper 70, Mr. T. Baker 50, Mr. Wheelock 40, Mr. Crocker 10, Mr. Boyden 90, Mr. Damrell 80 and Mr. Roberts 20. How much did all contribute? How many more dollars did Mr. J. M. Baker contribute than each of the others?

10. A man went by stage from Boston to Buffalo, a distance of 560 miles. The first day he travelled 60 miles, the second 40, the third 40, the fourth 50, the fifth 30, the sixth 50, the seventh 40, the eighth 70, the ninth 60, the tenth 20, the eleventh 30, the twelfth 20, and the thirteenth the remaining distance. How far was he from Buffalo at the close of each day's journey? How far did he travel the last day? How far was he from Boston at the close of each day's journey?

11. Myra, Hannah, Isabel, and Martha, each bought some ribbon at Mr. Crane's store. Myra bought a piece for 40 cents, Hannah bought a piece for 80 cents, Isabel for 80 cents, and Martha for 90 cents. How many cents' worth did they all buy? How many more cents' worth did they all buy than Myra? Than Hannah? Than Isabel? Than Martha? If, before buying the ribbon, Myra had 70 cents, Hannah had 120 cents, Isabel had 140 cents, and Martha had 160 cents, how many cents would each have left? How many would all have left? How many more did each spend than she had left?

12. Mr. Williams, a wealthy farmer, raised 500 barrels of apples, which he disposed of as follows, viz.:—He sold 40 barrels to one man, 50 to another, 70 to another, 20 to another, 60 to another, 80 to another, 40 to another, 70 to another, 30 to another, and he kept the rest for his own use. How many did he have left after the sale of each lot? How many did he keep for his own use? How many did he sell in all? He received 60 dollars for the first lot he sold, 80 dollars for the second, 90 dollars for the third, 40 for the fourth, 80 for the fifth, 100 for the sixth, 70 for the seventh, 100 for the eighth, and 60 for the ninth. How many dollars did he receive for all?

13. A gentleman had 7 rows of rose-bushes. In the first row there were 27, 7 of which died; in the second 20, in the third 9, one of which died; in the fourth 9, 2 of which died; in

the fifth 8, 3 of which died; in the sixth 30, and in the seventh 26, 6 of which died. How many of the bushes lived?

14. A gentleman journeying in his private carriage, travelled 35 miles on Monday, 7 miles on Tuesday, 8 on Wednesday, 20 on Thursday, 0 on Friday, and 10 on Saturday; how far did he travel during the week? The next Monday he travelled 17 miles, Tuesday 8, Wednesday 9, Thursday 10, Friday 9, Saturday 7. How far did he travel in the second week? How far in all? How many more miles in the first than in the second week?

15. A clerk in an English goods store sold to a lady a dress for 12 dollars, trimmings for 2 dollars, scarf for 1 dollar, table cover for 5 dollars, a cloak for 10 dollars, and 2 pairs of gloves at 1 dollar per pair. To another lady he sold a set of window-curtains, for 4 dollars, fixtures for 2 dollars, a piece of unbleached cotton for 5 dollars, some crash for 1 dollar. To a third, he sold 1 dress-pattern for 6 dollars, silk dress and trimmings for 20 dollars. What was the amount of his sales? Another clerk in the same store, sold 50 dollars' worth of goods, another 90 dollars' worth, another 80, and another 30. What was the amount sold by all?

## SECTION V.

**A.** 1. 12 tens + 3 units = 120 + 3, called ONE HUNDRED AND TWENTY-THREE, written 123.

2. 11 tens + 7 units = 110 + 7, called ONE HUNDRED AND SEVENTEEN, written 117.

3. 23 tens + 4 units = 230 + 4, called TWO HUNDRED AND THIRTY-FOUR, written 234.

4. 50 tens + 9 units = 500 + 9, called FIVE HUNDRED AND NINE, written 509.

5. 96 tens + 1 unit = 960 + 1, called NINE HUNDRED AND SIXTY-ONE, written 961.

6. 81 tens + 1 unit = 810 + 1, called EIGHT HUNDRED AND ELEVEN, written 811.

7. 66 tens + 6 units = 660 + 6, called SIX HUNDRED AND SIXTY-SIX, written 666.

8. 30 tens + 9 units = 300 + 9, called THREE HUNDRED AND NINE, written 309.

9. 52 tens + 5 units = 520 + 5, called FIVE HUNDRED AND TWENTY-FIVE, written 525.

10. 99 tens + 9 units = 990 + 9, called NINE HUNDRED AND NINETY-NINE, written 999.

Read the above numbers giving the value in tens and units separately.

**B.** 1. What do we call 1 hundred, 2 tens, and 3 units?
2. 4 hundreds, 7 tens, and 8 units?
3. 6 hundreds, 2 tens, and 1 unit?
4. 1 hundred, 1 ten, and 1 unit?
5. 5 hundreds, 1 ten, and 8 units?
6. 3 hundreds, 3 tens, and 3 units?
7. 9 hundreds, 0 tens, and 4 units?
8. 5 hundreds, 1 ten, and 1 unit?
9. 8 hundreds, 0 tens, and 3 units?
10. What do we call 16 tens and 4 units?
11. 24 tens and 3 units? 40 tens and 7 units?
12. 28 tens and 1 unit? 99 tens and 9 units?
13. 21 tens and 2 units? 60 tens and 8 units?
14. 33 tens and 4 units? 85 tens and 1 unit?
15. 10 tens and 8 units? 41 tens and 1 unit?
16. How many hundreds, tens, and units in 136? In 475?
17. In 854? in 121? in 690? in 728? in 307?
18. In 256? in 547? in 918? in 605? in 270?
19. In 777? in 111? in 234? in 432? in 324?

**C.** 1. What number will be represented by the character 2 in the first, 5 in the second, and 8 in the third place to the left from the decimal point?

2. By 9 in the first, 3 in the second, and 7 in the third place?

3. By 3 in the first, 0 in the second, and 6 in the third place?

4. By 7 in the first, 3 in the second, and 5 in the third place?
5. By 0 in the first, 1 in the second, and 4 in the third place?
6. By 1 in the first, 2 in the second, and 3 in the third place?
7. By 1 in the first, 3 in the second, and 2 in the third place?
8. By 2 in the first, 1 in the second, and 3 in the third place?
9. By 2 in the first, 3 in the second, and 1 in the third place?
10. By 3 in the first, 2 in the second, and 1 in the third place?
11. By 3 in the first, 1 in the second, and 2 in the third place?

**D.** 1. How is one hundred and forty-nine represented by figures?
2. Three hundred and eighteen? Nine hundred and seven?
3. Two hundred and fifty-nine?
4. Nine hundred and ninety-nine?
5. One hundred and eleven? Seven hundred and three?
6. Five hundred and five? Six hundred and sixty-one?
7. Four hundred and twenty-nine?
8. Six hundred and forty-three?
9. Three hundred and forty-six?
10. Four hundred and sixty-three?
11. Six hundred and thirty-four? One hundred and nineteen?

**E.** 1. 2 + 5 + 8? 22 + 5 + 8? 122 + 5 + 8?
2. 372 + 5 + 8? 912 + 5 + 8? 3 + 6 + 7?
3. 83 + 6 + 7? 183 + 6 + 7? 903 + 6 + 7?
4. 423 + 6 + 7? 9 + 4 + 7? 159 + 4 + 7?
5. 99 + 4 + 7? 189 + 4 + 7? 819 + 4 + 7?
6. 8 + 6 + 9? 88 + 6 + 9? 458 + 6 + 9?
7. 738 + 6 + 9? 888 + 6 + 9? 9 + 9 + 9?
8. 99 + 9 + 9? 739 + 9 + 9? 689 + 9 + 9?
9. 879 + 9 + 9? 7 + 8 + 4? 967 + 8 + 4?
10. 327 + 8 + 4? 597 + 8 + 4? 807 + 8 + 4?
11. 3 + 5 + 6? 333 + 5 + 6? 103 + 5 + 6?
12. 203 + 5 + 6? 913 + 5 + 6? 9 + 6 + 6?
13. 209 + 6 + 6? 199 + 6 + 6? 879 + 6 + 6?
14. 669 + 6 + 6?

**F.** 1. 16 — 7 — 5? 96 — 7 — 5? 436 — 7 — 5?
2. 23 — 8 — 7? 123 — 8 — 7? 463 — 8 — 7?
3. 903 — 8 — 7? 333 — 8 — 7? 20 — 6 — 8?
4. 100 — 6 — 8? 900 — 6 — 8? 640 — 6 — 8?
5. 810 — 6 — 8? 21 — 7 — 5? 321 — 7 — 5?
6. 111 — 7 — 5? 911 — 7 — 5? 411 — 7 — 5?
7. 54 — 9 — 6? 354 — 9 — 6? 814 — 9 — 6?
8. 304 — 9 — 6? 104 — 9 — 6? 32 — 5 — 9?
9. 232 — 5 — 9? 912 — 5 — 9? 302 — 5 — 9?
10. 822 — 5 — 9? 25 — 7 — 7? 725 — 7 — 7?
11. 925 — 7 — 7? 865 — 7 — 7? 385 — 7 — 7?
12. 21 — 5 — 7? 101 — 5 — 7? 811 — 5 — 7?

**G.** 1. 96 + 8 + 9 + 7 + 6 — 8 — 9 — 5 + 7 + 6?
2. 327 + 9 — 8 — 6 + 5 — 3 — 6 — 9 + 6 + 5?
3. 903 — 7 — 6 + 5 — 4 + 6 + 5 — 9 — 6 + 7?
4. 438 + 6 — 9 — 5 + 70 + 6 — 9 — 7 + 80?
5. 417 + 9 + 3 — 7 — 8 — 4 — 30 — 50 + 8?
6. 828 + 5 + 7 + 60 — 5 — 8 — 7 — 90 + 7?
7. 937 — 9 — 8 — 20 — 300 + 100 + 5 + 9?
8. 423 + 8 + 5 + 4 + 200 — 6 — 9?
9. 579 + 9 + 9 + 9 + 5 — 8 — 7 + 3 + 5?
10. 888 + 7 + 6 — 9 — 100 — 4 + 7 + 6 — 8?
11. 743 — 9 — 6 + 7 — 5 + 4 — 9 + 7 — 6?
12. 193 + 8 + 6 + 5 + 7 — 9 — 4 + 3 — 7 + 8 — 9 — 7?
13. 210 + 7 + 3 — 8 — 5 — 9 + 7 + 6 + 8 — 7 + 3?
14. 413 — 6 — 8 — 7 — 5 — 7 — 3 — 2 — 9 — 5 — 4 — 3?
15. 211 — 4 — 7 — 9 — 3 — 5 + 6 + 9 + 7 + 8 + 5 + 4?
16. 149 + 9 — 8 + 7 — 6 + 5 — 4 + 3 — 2 + 1 + 7 + 8?
17. 167 + 8 + 9 + 6 + 80 — 30 + 7 — 9 — 5 — 8 — 7?
18. 243 — 7 — 6 + 8 + 5 — 9 — 7 + 8 + 8 — 7 — 7 + 6?
19. 321 + 9 + 4 — 6 — 7 — 8 — 9 + 10 + 7 — 5 — 6 — 8 — 7 — 9?
20. 872 + 8 — 60 — 80 + 6 + 8 + 7 + 3 + 9 + 9 — 5 — 9 + 8 — 6?

**H.** 1. There were 479 apples on a tree, but 6 of them fell off, and 9 of them were stoned off by a mischievous boy. How many remained on the tree? 8 were blown off by the wind, and 7 were beaten off by the rain. How many then remained on the tree? The owner now picked all but 9 of them. How many did he pick? Of those he picked, he sold 8, and gave away 7. How many had he left?

2. A teacher asked his pupils to add 143, 7, 9, 8, 6, 4, 7, 9, 2, 5, 8, 8, 3, 7, 5, 2, 9, 9, 4, 3, 8, 6, 10, 3, and 7 together. They all answered correctly. What was their answer?

3. The same teacher asked a little girl named Susan to tell the remainders that would be obtained by subtracting successively from 428 the following numbers, viz.: 9, 7, 6, 3, 9, 4, 5, 8, 7, 6, 6, 6, 7, 7, 5, and 8. She answered right. What remainder did she name?

4. The same teacher wishing to obtain a new set of outline maps, the following sums were subscribed by his pupils for the purpose of purchasing them. By Benjamin Edmunds 40 cents, by Jacob F. Brown 50 cents, by Samuel W. Clapp 20 cents, by Edson W. Burr 30 cents, by Martha S. Price 20 cents, by Elizabeth Potter 20 cents, by Elizabeth Weston 9 cents, by Lemira A. Fish 8 cents, by Eunice M. Read 6 cents, by Annie S. Ireson 10 cents, by Francis W. Allen 7 cents, by Isabella E. Robinson 20 cents, by Caroline Tallant 8 cents, by Ira Copeland 9 cents, by Elisha Mann 9 cents, by Ira Moore 5 cents, by Edward Wellington 9 cents, by Bradford Tucker 80 cents, by Albert Boyden 90 cents, by John F. Barnard 60 cents, by Arthur Sumner 80 cents, by William James Potter 40 cents, by William Henry West 80 cents, by Lydia Dodge 30 cents, by Myra Sampson 20 cents, by Matilda Haven 20 cents, by Louise Cook 8 cents, by Ellis Holmes 9 cents, by Joseph Swain 3 cents, by Harry Keith 70 cents, and by Robert C. Metcalf 50 cents. How many cents were contributed by all?

5. A baker left 94 cents' worth of bread at one place, 6 cents' worth at another, 20 cents' worth at a third, 8 cents' worth at a fourth, 2 at a fifth, 30 at a sixth, 7 at a seventh, 4 at an eighth, 9 at a ninth, 40 at a tenth, 5 at an eleventh, 8 at a twelfth, 7 at a thirteenth, and 9 cents' worth at a store. How many cents' worth did he leave in all?

6. A lad, having 324 cents, spent 6 cents for pens, 8 for paper, 50 for a dictionary, 9 for a slate, 2 for India rubber, 9 for drawing paper, 20 for drawing patterns, 80 for a pair of shoes, 4 for apples, 6 for oysters, 2 for the mending of his sled, and 5 for shoe-strings. What had he left after each one of his purchases? He afterwards lost 9 cents. What did he have left? He then gave 7 cents to a poor woman, earned 8, found the 9

which he had lost, spent 4, earned 20, and gave 40 for the missionary cause. How much had he left?

7. Abby had 143 chestnuts, and Sarah had 135; Abby gave 7 of hers to Sarah. How many had she left? How many did Sarah have? Abby ate 3 of hers, and Sarah ate 1 of hers. How many did each have left? Abby gave away 6, and Sarah gave away 7. How many did each now have? How many would each have if Sarah should now give 8 to Abby?

8. A man bought a chaise for 136 dollars, and gave 6 dollars for having it repaired, and 9 dollars for having it varnished. How much did it cost him in all? He sold it for 9 dollars less than the cost. What did he sell it for? With the money he received for the chaise he bought a horse. He paid 7 dollars for keeping him, and then sold him for 8 dollars more than he had paid for the horse and his keeping. What did he sell him for?

9. A grain dealer had 678 bushels of oats. He afterwards bought 9 bushels at one time, 7 bushels at another, 8 at another, 6 at another, and 9 at another. He sold 6 bushels to one man, 9 bushels to another, and 8 bushels to another, bought 7 bushels from another, 5 bushels from another, and 8 bushels from another, and then sold 6 bushels to Mr. Mason, 9 bushels to Mr. Marsh, 8 bushels to Mr. Sampson, 5 bushels to Mr. Ford, 8 bushels to Mr. Cobb, 9 bushels to Mr. Cleaveland, 7 bushels to Mr. Wilkinson, and 8 bushels to Mr. Richardson. How many bushels did he then have?

10. A peddler, who lived in a town 211 miles west of N. York, left home Dec. 1, 1851, on a peddling excursion. He travelled 8 miles towards the east on the 1st day of the month, 9 miles towards the west on the 2d, 8 miles towards the west on the 3d, 5 miles towards the west on the 4th, 9 miles towards the east on the 5th, 8 miles towards the east on the 6th, and, the 7th day being Sunday, he did not travel. He travelled 9 miles towards the east on the 8th, 8 miles towards the west on the 9th, 7 miles towards the east on the 10th, 6 miles towards the east on the 11th, 9 miles towards the west on the 12th, 8 miles towards the west on the 13th, 9 miles towards the east on the 15th, 5 miles towards the east on the 16th, 7 miles towards the west on the 17th, 4 miles towards the east on the 18th, 6 miles towards the west on the 19th, 8 miles towards the west on the 20th, 7 miles towards the east on the 22d, 6 miles towards the east on the 23d, 5 miles towards the east on the 24th, 9 miles towards the west on the 25th, 9 miles towards the west on the 26th, 6 miles towards the west on the 27th, 9 miles towards the

east on the 29th, 8 miles towards the east on the 30th, and 9 miles towards the east on the 31st. How far was he from N. York at the end of the month? How far was he from N. York when nearest to it? When the farthest from it? In which direction and how far from home was he on the 1st Sunday in the month? on the 2d? on the 3d? on the 4th? How many miles did he travel in all?

11. He had 113 dollars with him when he left home. On the 1st day of the month he received 9 dollars and paid out 2, on the 2d he received 8 dollars, on the 3d he received 7, on the 4th he paid out 10, on the 5th he received 8 and paid out 3, on the 6th he received 10, on the 8th he received 9 and paid out 4, on the 9th he received 7, on the 10th he paid out 5, on the 11th he received 9 and paid out 4, on the 12th he received 8, on the 13th he received 7, on the 15th he received 5 and paid out 9, on the 16th he paid out 7, on the 17th he paid out 9 and received 1, on the 18th he received 8, on the 19th he received 10, on the 20th he received 10 and paid out 2, on the 22d he received 9 and paid out 5, on the 23d he received 8, on the 24th he received 10, on the 25th he received 9, on the 26th he received 7, on the 27th he paid out 4, on the 29th he paid out 9 and received 9, on the 30th he received 10, and on the 31st he received 8 and paid out 6. How many dollars did he have at the end of the month? How many dollars did he receive during the month? How many did he pay out?

## SECTION VI.

**A.** 1. 20 + 70 + 80? 22 + 70 + 80? *Solution.* 22 + 70 = 92, + 80 = 172.

**Note.** It should be borne in mind that in *all abstract examples*, the pupil should be taught *finally* to solve the questions by naming only the results of indicated operations.

2. 50 + 90 + 20? 54 + 90 + 20? 154 + 90 + 20?
3. 80 + 30 + 70? 83 + 30 + 70? 483 + 30 + 70?
4. 20 + 70 + 80? 25 + 70 + 80? 325 + 70 + 80?
5. 30 + 80 + 70? 836 + 80 + 70? 536 + 80 + 70?
6. 35 + 80 + 20 + 90 + 20 + 40 + 70?

7. 67 + 30 + 60 + 90 + 20 + 20 + 30?
8. 28 + 60 + 80 + 30 + 50 + 10 + 90?
9. 437 + 80 + 50 + 100 + 60 + 90 + 60?
10. 11 + 90 + 80 + 30 + 60 + 30 + 50?
11. 347 + 80 + 60 + 70 + 80 + 90 + 200?
12. 182 + 30 + 70 + 30 + 60 + 20 + 90?
13. 211 + 80 + 20 + 80 + 70 + 300 + 50?
14. 23 + 70 + 20 + 50 + 80 + 90 + 200?
15. 726 + 40 + 90 + 30 + 20 + 30 + 60?
16. 149 + 70 + 30 + 80 + 200 + 70 + 50?
17. 237 + 40 + 90 + 60 + 20 + 70 + 80?

**B.** 1. 170 — 80 — 30? 178 — 80 — 30? *Solution.* 178 — 80 = 98, — 30 = 68.

2. 140 — 60 — 20? 143 — 60 — 20? 643 — 60 — 20?
3. 220 — 60 — 70? 223 — 60 — 70? 523 — 60 — 70?
4. 610 — 50 — 80? 613 — 50 — 80? 213 — 50 — 80?
5. 338 — 40 — 70? 638 — 40 — 70? 588 — 40 — 70?
6. 428 — 30 — 90? 528 — 30 — 90? 218 — 40 — 70?
7. 821 — 50 — 60 — 90 — 50 — 60 — 90?
8. 411 — 70 — 40 — 60 — 30 — 70 — 40?
9. 964 — 80 — 50 — 70 — 80 — 50 — 70?
10. 629 — 50 — 40 — 80 — 60 — 20 — 60?
11. 931 — 50 — 80 — 70 — 60 — 50 — 70?
12. 314 — 30 — 20 — 50 — 30 — 60 — 70?

**C.** 1. 124 + 65? *Solution.* 124 + 60 = 184, + 5, = 189; or 124, 184, 189.

2. 47 + 89? 63 + 85? 87 + 26? 147 + 83?
3. 196 + 87? 349 + 84? 364 + 87? 459 + 37?
4. 684 + 69? 545 + 83? 397 + 48? 436 + 93?
5. 82 + 74 + 63? 97 + 82 + 54? 131 + 44 + 83?
6. 824 + 37 + 25? 637 + 51 + 88? 128 + 75 + 83?
7. 246 + 84 + 91? 425 + 75 + 83? 87 + 54 + 59?
8. 286 + 94 + 73? 823 + 77 + 62? 229 + 47 + 88?
9. 143 + 78 + 64? 732 + 88 + 97? 245 + 38 + 73?
10. 837 + 49 + 91? 837 + 54 + 98? 823 + 97 + 48?

11. 87 + 44 + 56 ? 98 + 73 + 87 ? 561 + 83 + 24 ?
12. 111 + 97 + 63 ? 125 + 88 + 43 ? 323 + 28 + 69 ?
13. 756 + 57 + 59 ? 666 + 66 + 66 ? 777 + 77 + 77 ?
14. 448 + 74 + 23 ? 446 + 46 + 46 ? 378 + 78 + 78 ?
15. 219 + 87 + 19 ? 218 + 46 + 43 ?

**D.** 1. 224 — 49 ? *Solution.* 224 — 40 = 184, — 9 = 175, *Ans.* Or 224, 184, 175, *Ans.*

2. 178 — 96 ? 85 — 37 ? 418 — 73 ? 93 — 28 ?
3. 146 — 85 ? 311 — 47 ? 964 — 86 ? 321 — 85 ?
4. 417 — 38 ? 127 — 58 ? 481 — 37 ? 512 — 63 ?
5. 431 — 82 — 76 ? 428 — 37 — 49 ? 931 — 82 — 56 ?
6. 357 — 59 — 83 ? 191 — 83 — 79 ? 847 — 68 — 72 ?
7. 438 — 57 — 42 ? 198 — 43 — 64 ? 257 — 49 — 83 ?
8. 672 — 83 — 54 ? 711 — 48 — 17 ? 93 — 27 — 38 ?
9. 839 — 18 — 16 ? 321 — 87 — 16 ? 813 — 48 — 67 ?
10. 325 — 87 — 29 ? 761 — 45 — 99 ? 832 — 99 — 99 ?
11. 620 — 54 — 37 ? 373 — 77 — 77 ? 507 — 88 — 36 ?
12. 196 — 73 — 85 ? 401 — 86 — 37 ? 922 — 46 — 57 ?

**E.** 1. 48 + 57 + 23 + 64 ? 157 + 86 + 97 + 31 ?

2. 591 + 87 + 57 + 38 ? 611 — 28 + 87 — 99 ?
3. 421 + 64 — 83 — 67 ? 117 + 17 + 17 — 19 ?
4. 432 + 87 + 59 — 69 ? 836 — 79 + 59 + 47 ?
5. 517 — 93 + 21 — 48 ? 437 — 98 + 56 — 77 ?
6. 221 — 45 — 97 + 85 ? 527 + 38 — 63 — 72 ?
7. 814 — 97 — 86 — 76 ? 541 + 64 — 78 — 67 ?
8. 941 — 87 — 96 — 63 ? 297 — 39 — 28 — 19 ?
9. 407 — 69 + 43 — 76 ? 423 + 97 — 85 — 87 ?
10. 731 — 49 — 58 + 23 ? 854 — 78 + 67 — 69 ?
11. 27 + 88 + 95 — 13 ? 551 + 37 — 86 — 87 ?

**F.** Add the numbers in the following columns.

| First. | Second. | Third. | Fourth. | Fifth. | Sixth. | Seventh. | Eighth. |
|---|---|---|---|---|---|---|---|
| 29 | 83 | 19 | 84 | 94 | 67 | 94 | 66 |
| 84 | 64 | 17 | 67 | 82 | 58 | 75 | 88 |
| 96 | 99 | 46 | 53 | 28 | 39 | 63 | 99 |
| 51 | 25 | 78 | 41 | 46 | 62 | 85 | 47 |
| 78 | 37 | 87 | 29 | 64 | 87 | 97 | 89 |
| 43 | 72 | 94 | 13 | 37 | 54 | 58 | 95 |
| 27 | 69 | 49 | 17 | 73 | 66 | 83 | 16 |

**G.** 1. 367 + 454? *Solution.* 367 + 400 = 767, + 50 = 817, + 4 = 821, *Ans.* Or 367, 767, 817, 821, *Ans.*

2. 516 + 485? 327 + 482? 527 + 275? 884 + 116?

3. 217 + 587 + 111? 174 + 227 + 500? 325 + 144 + 507?

Add the numbers in the following columns as by above solution.

| Fourth. | Fifth. | Sixth. | Seventh. | Eighth. | Ninth. |
|---|---|---|---|---|---|
| 254 | 244 | 187 | 214 | 509 | 222 |
| 327 | 115 | 256 | 199 | 170 | 333 |
| 111 | 312 | 199 | 318 | 177 | 277 |
| 178 | 328 | 335 | 216 | 107 | 188 |

**H.** 1. 617 — 329? *Solution.* 617 — 300 = 317, — 20 = 297, — 9 = 288, *Ans.* Or 617, 317, 297, 288, *Ans.*

2. 925 — 745? 724 — 295? 427 — 248? 323 — 166?
3. 777 — 288? 629 — 333? 678 — 599? 572 — 257?
4. 541 — 279? 327 — 198? 369 — 273? 911 — 675?
5. 631 — 275? 382 — 193? 824 — 675? 621 — 328?
6. 547 — 211? 932 — 595? 721 — 483? 264 — 327?
7. 618 — 479? 234 — 198? 591 — 287? 613 — 257?
8. 876 — 183? 841 — 757? 922 — 333? 523 — 275?
9. 888 — 299? 532 — 199? 643 — 274? 518 — 279?

**I.** 1. What number must be added to 50 that the sum shall equal 100? What number must be added to 70?

2. To 30? to 90? to 10? to 25? to 85? to 15? to 94?

3. To 84? to 74? to 14? to 24? to 44? to 81? to 1?

4. To 5? to 8? to 78? to 16? to 46? to 37? to 36?

5. To 64? to 41? to 73? to 19? to 57? to 43?

6. What number must be added to 400 that the sum shall equal 1000? What number must be added to 600?

7. To 50? to 80? to 150? to 450? to 891? to 673?

8. To 973? to 273? to 456? to 225? to 250? to 750?

9. To 825? to 125? to 888? to 666? to 384? to 575?

10. To 875? to 333? to 411? to 673? to 527? to 425?

11. To 897? to 138? to 279? to 842? to 147? to 853?

**J.** 1. There are 58 towns in Worcester county, Mass., 50 in Middlesex, 31 in Berkshire, 30 in Essex, 26 in Franklin, 23 in Hampshire, 22 in Plymouth, 22 in Norfolk, 20 in Hampden, 19 in Bristol, 13 in Barnstable, 3 in Suffolk, 3 in Dukes, and 1 in Nantucket. How many towns in all these counties?

2. The number of Representatives for the valuation year, from Suffolk county is 47, from Essex is 49, from Middlesex is 68, from Worcester is 63, from Berkshire is 33, and from Norfolk is 30. How many Representatives from the six counties?

3. A Vermont farmer had 17 cows, 24 oxen, 59 sheep, and 19 horses. How many animals had he? He sold 2 cows for 67 dollars, 1 yoke of oxen for 85 dollars, 17 sheep for 33 dollars, and 2 horses for 99 dollars. How many dollars did he receive for them? How many animals did he sell? How many had he left?

4. The Boston Insurance Company owns 190 shares in the Connecticut River Railroad, 150 in the Boston and Providence Railroad, 20 in the Boston and Lowell Railroad, 150 in the Taunton Branch Railroad, and 100 in the Old Colony Railroad. How many shares are owned by the Company?

5. The Tremont Insurance Company, at Boston, owns bank stock as follows: In the Granite Bank 198 shares, in the City Bank 47 shares, in the Atlantic 10, in the Atlas 30, in the Market 13, in the North 55, in the Merchants' 27, in the Shoe and Leather Dealers' 30, in the Union 18, in the State 14, in the Tremont 216, in the New England 123, and in the Grocers' 50. How many shares do they own? They own 50 shares in the Taunton Branch Railroad, 100 in the Boston and Providence, and 76 in the Boston and Worcester. What is the number of shares of railroad stock?

6. The Washington Insurance Company owns 100 shares in

the Atlas Bank, 50 in the City Bank, 50 in the Hamilton, 55 in the Massachusetts, 50 in the Market, 75 in the North, 129 in the New England, 26 in the Suffolk, 142 in the State, and 76 in the Union. Required the number of shares held in all the banks.

7. The Marblehead Marine Insurance Company owns 200 shares in the Suffolk Bank, 150 in the Globe, 150 in the New England, 100 in the Eagle, 100 in the Columbian, 40 in the City, 45 in the Atlantic, 25 in the North, and 120 in the Marblehead. How many shares does the Company hold in these banks?

8. The Essex Insurance Company owns 185 shares in Naumkeag Bank, 142 in the Commercial, 144 in the Exchange, 153 in the Asiatic, 22 in the Mercantile, 9 in the Salem, 74 in the Merchants', 75 in the North, and 20 in the Shawmut. Required the number of shares held in all these banks.

9. The Old Colony Insurance Company owns 35 shares in the Wareham Bank, 40 in the Railroad Bank, 15 in the Union Bank, 20 in the Market Bank, 15 in the New England, 24 in the Old Colony Bank, 25 in the Merchants', 21 in the Plymouth, 10 in the Massachusetts, and 20 in the Boston. How many shares are held by the Company in all these banks?

10. In sinking a water shaft in a coal mine in Rhode Island, the following depths were penetrated: 9 feet of sand and gravel, 12 of dark-colored slate, 23 of graywacke, 41 of soft black slate, 50 of hard brown slate, 10 of gray slate, 15 of very hard brown slate, 12 of gray freestone, and 4 of coal. What was the distance from the surface of the ground to each of the above rocks? How deep was the shaft sunk?

11. The highest point of land in Waltham is 570 feet in height. How much higher is this hill than Prospect Hill, in the same place, which has an elevation of 470 feet? Than Bear Hill, which has an elevation of 510 feet? How much higher is Bear Hill than Prospect Hill?

12. Mr. Allen had 25 dollars, Mr. Abbot paid him 17 dollars, and Mr. Dickerson paid him 19 dollars; he then paid a debt of 31 dollars, and bought some flour for 11 dollars; he spent the remainder of his money for broadcloth, which he sold for 35 dollars, and with the money gained on the broadcloth he bought 20 bushels of corn, which he sold for 24 dollars. How much money did he then have? How much more than he had at first?

13. Mr. Kaime bought 17 bushels of quinces for 35 dollars. He sold 9 bushels of them to Mr. Ballou for 26 dollars, and the

remainder to Mr. Cook for 33 dollars. He then spent all his money for 50 bushels of apples which he sold for 75 dollars. With the money gained on the apples he bought sugar, which he sold for 11 dollars more than he gave for it. How much money had he then? How much did he gain on his quinces, apples, and sugar? How much more on the apples than on the quinces and sugar?

14. A man carried 5 loads of potatoes to market. There were 23 bushels in the first load, 19 in the second, 25 in the third, 27 in the fourth, and 29 in the fifth. How many bushels were there in all? He received 18 dollars for the first load, and 100 dollars for them all. How much did he receive for the last four loads? He received 14 dollars for the second load. How much did he receive for the last three? He received 21 dollars for the third load. How much did he receive for the last two? He received 23 dollars for the fourth load. How much did he receive for the last?

15. A farmer, who raised 134 bushels of corn, sold 97 bushels, and kept the rest for his own use. How many bushels did he keep? He raised so many oats that after selling 96 bushels to one man, 43 to another, and 87 to another, he had 58 bushels left. How many bushels of oats did he raise? He raised 124 tons of hay, of which he sold 37 tons. How many tons did he keep? He gathered 827 bushels of apples, of which 136 bushels were cider apples, 87 bushels were early apples, 128 bushels were fall apples, and the remainder were winter apples. How many were winter apples? From the cider apples he made 20 barrels of cider; he sold 79 bushels of early apples, 117 bushels of fall apples, and 443 bushels of winter apples, using the rest in his family. How many bushels of each kind did he use? How many bushels of apples did he sell? He raised 234 bushels of potatoes, of which he used 38 bushels, selling the rest. How many did he sell? He received 73 dollars for the corn, 93 dollars for the oats, 628 dollars for the hay, 75 dollars for the early apples, 87 dollars for the fall apples, 341 dollars for the winter apples, and 150 dollars for the potatoes. How much did he receive for all he sold?

16. A news-boy sold on Monday some papers for 96 cents, which cost him 54 cents; on Tuesday for 87 cents, which cost him 39 cents, and on Wednesday for 93 cents, which cost him 49 cents. How many cents did he gain on what he sold in three days?

17. Jemmy Daily had 17 cents with which he purchased some papers, which he sold so as to gain 14 cents on them.

With the money he thus received he purchased some more papers, which he sold so as to gain 28 cents on them. After giving his mother 9 cents to buy some bread with, he spent the rest of his money for papers, which he sold so as to gain 37 cents on them. He now gave 43 cents to his mother, keeping the rest to buy some more papers with. How many cents did he keep?

18. A man started from Boston for the purpose of seeking his fortune, taking with him 675 dollars. He first went to New York, from thence to Philadelphia, and there took ship for San Francisco. His fare from Boston to New York was 4 dollars, and from New York to Philadelphia 3 dollars. While stopping in Philadelphia he was robbed of his valise, containing 100 dollars. His passage to San Francisco cost him 225 dollars. After remaining in San Francisco 1 week and 2 days at an expense of 27 dollars, he started for the mines. He worked at the "wet diggings" 30 days, and at the "dry diggings" 15 days. He sold the gold he got at the "wet diggings" for 375 dollars, and that which he got at the "dry diggings" for 87 dollars. The expenses of his trip to the mines were 117 dollars. From San Francisco he went to Calcutta, paying 50 dollars for his passage, and immediately upon his arrival he went into business. At the end of three months his health became so poor that he was obliged to give up business and return to Boston. His profits while in Calcutta were 347 dollars. His passage to Boston cost him 115 dollars. How much money had he when he arrived at Boston? Had he more or less than when he left Boston, and how much?

## SECTION VII.

**A.** All nations, excepting perhaps the most barbarous, have some kind of money; but each nation has a system peculiar to itself, and generally differing from every other in its denominations, coins, &c. The people of the United States reckon money in dollars and cents; the English reckon it in pounds, shillings and pence; and the French, in francs and centimes.

So, too, each nation has a peculiar system of weights and measures, some of the most important of which are illustrated in the following

## TABLES OF MONEY, WEIGHTS, AND MEASURES.

**B.** The money of the United States is called Federal Money.

### TABLE OF FEDERAL MONEY.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

The coins of the United States are,—the dollar; the half-dollar, or fifty-cent piece; the quarter-dollar, or twenty-five-cent piece; the dime, or ten-cent piece; the half-dime, or five-cent piece; the three-cent piece; the cent; the eagle, or ten-dollar piece; the double eagle, or twenty-dollar piece; the half-eagle, or five-dollar piece; and the quarter-eagle, worth $2\frac{1}{2}$ dollars.

The dollar is coined both of gold and silver; the coins worth more than a dollar are of gold, and the others, with the exception of the cent, are of silver. The cent is of copper.

**C.** The money used in England is called English, or Sterling Money.

### TABLE OF STERLING MONEY.

4 farthings = 1 penny.
12 pence = 1 shilling.
20 shillings = 1 pound.

When numbers expressing values in Sterling money, are used, the character £ marks the pounds; s. the shillings; d. the pence; and qr. the farthings.

*Illustration.* 27 pounds, 18 shillings, 5 pence, and 3 farthings, may be written thus: 27£, 18s. 5d. 3 qrs.; or thus: $\overset{£}{27}$, $\overset{s.}{18}$, $\overset{d.}{5}$, $\overset{qrs.}{3}$. Farthings are also frequently expressed as fourths of a penny. Thus, 7d. 3qrs. = $7\frac{3}{4}$d., and both forms may be read as 7 pence and 3 farthings.

The coin representing the pound is called the Sovereign. Its value in United States money varies from 4 dollars and 83 cents, to 4 dollars and 86 cents, but is usually about 4 dollars and 84 cents. There are several other coins used in England, of which we will mention only two, viz.: the Guinea, or 21 shilling piece, and the Crown, or 5 shilling piece.

**D.** Almost all articles, except gold, silver, and jewels, are weighed by what is called Avoirdupois Weight.

TABLE OF AVOIRDUPOIS WEIGHT.

16 drams = 1 ounce.
16 ounces = 1 pound.
28 pounds = 1 quarter.
4 quarters = 1 hundred weight.
20 hundred weight = 1 ton.

The abbreviations made use of in this weight, are T. for ton, cwt. for hundred weight, qr. for quarter, lb. for pound, oz. for ounce, and dr. for dram. For example: 5 T. 17 cwt. 3 qr. 13 lb. 8 oz. 7 dr. = 5 tons, 17 hundred weight, 3 quarters, 13 pounds, 8 ounces, and 7 drams.

In several of the States of the Union, laws have been made fixing the hundred weight at 100 lb., the ton at 2000 lb., and the quarter at 25 lb., but in the standard of the United States government, and in that of Great Britain, the weights are reckoned as in the table.

**E.** The weight used in weighing Gold, Silver, and Precious Stones, is called Troy Weight. This weight is also used in philosophical experiments.

TABLE OF TROY WEIGHT.

24 grains = 1 pennyweight.
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

In this weight, lb. stands for pound, oz. for ounce, dwt. for pennyweight, and gr. for grain. Illustration: — 13 pounds, 7 ounces, 18 pennyweights, 23 grains, may be expressed thus: 13 lb. 7 oz. 18 dwt. 23 gr.; or thus: $\overset{\text{lb.}}{13}, \overset{\text{oz.}}{7}, \overset{\text{dwt.}}{18}, \overset{\text{gr.}}{23}$.

**F.** The weight used in compounding or mixing medicines, is called Apothecaries' Weight. Physicians write their prescriptions in this weight; but medicines are bought and sold by Avoirdupois Weight.

TABLE OF APOTHECARIES' WEIGHT.

20 grains = 1 scruple.
3 scruples = 1 dram.
8 drams = 1 ounce.
12 ounces = 1 pound.

To mark the denominations of this weight, we use the following characters, viz.:—℔ for pound, ℥ for ounce, ʒ for dram, ℈ for scruple, and gr. for grain. For example: 5 ℔ 6 ℥ 4 ʒ 2 ℈ 17 grs., would be read as 5 pounds, 6 ounces, 4 drams, 2 scruples, and 17 grains.

**G.** The only difference between Troy Weight and Apothecaries' Weight is, that in the former the ounce is divided into pennyweights and grains, while in the latter it is divided into drams, scruples, and grains. The pound, ounce, and grain, are the same in both weights.

The value of denominations of the same name in Avoirdupois and Troy Weights, differs very materially, as may be seen from the following Table, which shows the value in Troy grains of each denomination we have given in the preceding tables of weights.

1 lb. Av. = 7000 gr. Troy.
1 " Tr. = 1 ℔. = 5760 " "
1 oz. Av. = $437\frac{1}{2}$ " "
1 " Tr. = 1 ℥. = 480 " "
1 dr. Av. = $27\frac{11}{32}$ " "
1 ʒ = 60 " "
1 ℈ = 20 " "
1 dwt. = 24 " "
1 gr. Ap. = 1 " "

From the above table, we should find by calculation, that 144 lb. Av. = 175 lb. Tr., and 192 oz. Av. = 175 oz. Tr. Therefore, 1 lb. Av. = $\frac{175}{144}$ or $1\frac{31}{144}$ lb. Tr., and 1 oz. Av. = $\frac{175}{192}$ oz. Tr.

**H.** Distances in any direction are measured in the denominations contained in the following

TABLE OF LONG MEASURE.

12 inches = 1 foot.
3 feet = 1 yard.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, = 1 rod, or pole.
40 rods = 1 furlong.
8 furlongs = 1 mile.
3 miles = 1 league.

In this measure, le. stands for league, m. for mile, f. for furlong, rd. for rod, yd. for yard, ft. for foot, and in. for inches.

Surveyors usually measure distances by means of a chain 4 rods in length, called Gunter's chain, or the Surveyor's chain. This chain contains 100 equal links; 25 links will, therefore, equal 1 rod, and 1 link will equal $7\frac{23}{25}$ inches.

**I.** The following measure is used for measuring cloth, ribbons, &c.

TABLE OF CLOTH MEASURE.

$2\frac{1}{4}$ inches = 1 nail.
4 nails = 1 quarter.
4 quarters = 1 yard.

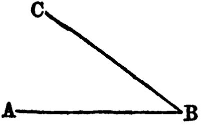
In this measure, yd. stands for yard, qr. for quarter, and na. for nail.

The yard and inch are the same in length as the yard and inch Long Measure.

**J.** Square Measure is used for measuring surfaces. As true ideas of the nature of the right angle, rectangle, and square, are essential to a just appreciation of this measure, we insert the following definitions and illustrations, leaving it with the teacher to give such others as he may deem necessary.

When two lines meet at a point, their difference in direction is called the angle of the two lines. The point where they meet is called the vertex of the angle.

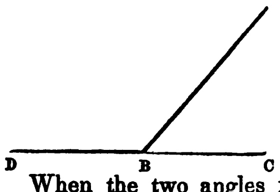
Fig 1.



In this figure the lines marked B A and B C form an angle whose vertex is at B.

In reading an angle, the letter at the vertex is always made the middle one. The angle in figure 1 may be read, either as the angle A B C, or as the angle C B A.

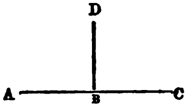
Fig. 2.



In this figure there are two angles, viz.: D B A, and A B C. The first is the difference in direction of the two lines A B and D B, and the second is the difference in direction of the two lines A B and B C. It is evident that the first is larger than the second.

When the two angles formed by one straight line meeting another are equal to each other, they are called right angles, and the two lines are said to be perpendicular to each other.

Fig. 3.



Suppose the straight line D B has met the straight line A C, so as to make the adjacent angles A B D and D B C equal to each other; then A B D and D B C will each of them be right angles, and D B and A C will be perpendicular to each other.

An angle greater than a right angle is called an obtuse (blunt) angle, and one less than a right angle is called an acute (sharp) angle. In Fig. 2, D B A is an obtuse angle, and A B C is an acute angle. In Fig. 1, A B C is an acute angle.

A four-sided figure having all its angles right angles is called a rectangle.

A rectangle having all its sides equal is called a square. A square, then, has four equal sides and four equal angles. Figures 4 and 5 represent rectangles. Figure 5 also represents a square.

Fig. 4.

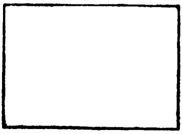
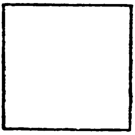


Fig. 5.



If a square measures a foot on each side, it is called a square foot; if it measures a yard on a side, it is called a square yard, &c. &c.

All surfaces are measured by Square Measure, that is, by the number of squares of a given size to which they are equivalent. Thus, a surface contains 5 square feet, when it is equivalent to 5 squares, each measuring 1 foot on a side.

TABLE OF SQUARE MEASURE.

| | | |
|---|---|---|
| 144 square inches | = | 1 square foot. |
| 9 square feet | = | 1 square yard. |
| 30¼ square yards, or<br>272¼ square feet | = | 1 square rod. |
| 40 square rods | = | 1 rood. |
| 4 roods | = | 1 acre. |
| 640 acres | = | 1 mile. |

In this measure, Sq. M. stands for square mile, A. for acre, R. for rood, sq. rd. for square rod, sq. yd. for square yard, sq. ft. for square foot, and sq. in. for square inch.

**K.** A solid is a body which has length, breadth, and thickness.

A cube is a rectangular solid, whose length, breadth, and height are equal. It may also be defined as a solid which is bounded by six equal squares.

A cube 1 foot long, 1 foot wide, and 1 foot high, would be a cubic foot. A cube 1 yard long, 1 yard wide, and 1 yard high, would be a cubic yard.

The solid contents of bodies are measured by cubic measure, *i. e.*, by the number of cubes of a given size which the bodies contain, or to which they are equivalent.

### TABLE OF CUBIC MEASURE.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
16 cubic feet = 1 cord foot.
8 cord feet, or 128 cubic feet, } = 1 cord of wood.

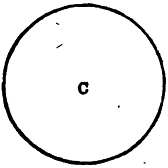
In this measure, C. stands for cord, Cd. ft. for cord foot, cu. ft. for cubic foot, cu. yd. for cubic yard, and cu. in. for cubic inch.

**L.** Circular or Angular Measure is used to measure angles, and the circumferences of circles.

A circle is a surface bounded by a curved line which is everywhere equally distant from a point within called the centre. The boundary-line is called the circumference of a circle.

Fig. 6 represents a circle of which C is the centre.

Fig. 6.



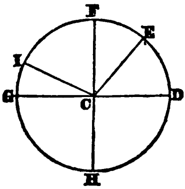
The distance from the centre of a circle to the circumference is called the radius of the circle.

The distance from a point on one side of a circle through the centre to a point on the opposite side is called the diameter of the circle. Any portion of the circumference is called an arc. Every circumference of a circle, whether the circle be large or small, is supposed to be divided into 360 equal parts called degrees. Each degree is divided into 60 equal parts called minutes, and each minute into 60 equal parts called seconds. A degree is to be regarded simply as the 360th part of the circumference of the circle considered. Hence it is obvious that its length, and that of its subdivisions, must vary with the size of the circle.

If from the vertex of an angle as a centre, we should draw a circumference, some portion of that circumference would be included between the sides of the angle. The larger the angle is, the larger will be the arc included between its sides, and the smaller the angle is, the smaller will be the included arc. Since the angle and the arc thus vary with each other, we take the arc as the measure of the angle. If the arc contains 50 degrees, the angle is one of 50 degrees, &c.

Figure 7 may illustrate this.

Fig. 7.



In this figure let C be the centre of the circle and the vertex of the several angles, and let F H and G D be lines perpendicular to each other, and C E and C I be lines drawn at random. Then the arc E D is the measure of the angle E C D; the arc D F, of the angle D C F; the arc D I, of the angle D C I; the arc E F, of the angle E C F, &c.

Since F H and G D are perpendicular to each other, the angles F C D, D C H, H C G and G C F are all right angles, and each of them must include $\frac{1}{4}$ of the angular space about the point C. Therefore, the arc included between the sides of a right angle equals $\frac{1}{4}$ of the circumference, and the measure of a right angle is $\frac{1}{4}$ of 360 degrees, which equals 90 degrees. The measure of an acute angle is less than 90 degrees, and that of an obtuse angle more than 90 degrees.

### TABLE OF CIRCULAR, OR ANGULAR MEASURE.

60 seconds = 1 minute.
60 minutes = 1 degree.
360 degrees = 1 circumference.

Degrees are marked by the character °, minutes by ′, seconds by ″: Thus, 13° 27′ 49″ = 13 degrees, 27 minutes and 49 seconds.

An arc of 90° is called a Quadrant.

A Sign is an astronomical measure of 30°.

**M.** Dry Measure is used for measuring all kinds of grain, beans, nuts, salt, &c.

### TABLE OF DRY MEASURE.

2 pints = 1 quart.
8 quarts = 1 peck.
4 pecks = 1 bushel.

The Chaldron of 36 bushels is sometimes used in measuring coals. ch. stands for chaldron, bu. for bushel, pk. for peck, qt. for quart, and pt. for pint.

The bushel contains $2150\frac{2}{5}$ cubic inches. The quart must, therefore, contain $67\frac{1}{5}$ cubic inches.

**N.** All kinds of liquids are measured by Liquid Measure.

### LIQUID MEASURE.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.

In this measure gal. stands for gallon, qt. for quart, pt. for pint and gi. for gill.

The hogshead of 63 gallons is used in estimating the contents of reservoirs or other large bodies of water, but in all other cases, the term hogshead is not a definite measure. Casks containing from 50 or 60 to 100 or 200 gallons are called hogsheads.

A barrel of *cider* is usually reckoned at $31\frac{1}{2}$ gallons.

The gallon contains 231 cubic inches. The *beer gallon* is sometimes used for measuring milk, beer and ale.

The beer gallon contains 282 cubic inches. The beer quart must therefore contain $70\frac{1}{2}$ cubic inches.

The following table will be convenient for use in comparing Dry, Liquid, and Beer Measures:

1 qt. Dry Measure = $67\frac{1}{5}$ cubic inches.
1 qt. Liquid Measure = $57\frac{3}{4}$ cubic inches.
1 qt. Beer Measure = $70\frac{1}{2}$ cubic inches.

### **O.** TABLE OF TIME.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
$\left.\begin{array}{l}365\frac{1}{4}\text{ days or}\\ 52\text{ weeks and }1\frac{1}{4}\text{ days}\end{array}\right\} = 1$ year.

The year is divided into 12 months, which differ somewhat in length. In this measure yr. stands for year, mo. for month, wk. for week, da. for day, h. for hour, m. for minute, and sec. for second.

To avoid the inconvenience of reckoning $\frac{1}{4}$ of a day with each year, every fourth year (called *Leap year*) is reckoned at 366 days and the others are reckoned at 365 days. A leap year may always be known by this, viz.:—Its number can be divided by 4. Thus we know that 1852 is a leap year, because 1852 can be divided by 4 without a remainder.

The year in reality contains but 365 days, 5 h. 48 m. 48 sec. so that by reckoning 365$\frac{1}{4}$ days we make a slight error each year which in 100 years amounts to about 1 day. The Centennial years are not, therefore, reckoned as leap years unless the number of the year be divisible by 400. Thus the year 1900 will not be a leap year; but the year 2000 will be.

### TABLE OF THE MONTHS.

January has 31 days.
*February has 28 days.
March has 31 days.
April has 30 days.
May has 31 days.
June has 30 days.
July has 31 days.
August has 31 days.
September has 30 days.
October has 31 days.
November has 30 days.
December has 31 days.

### P. MISCELLANEOUS.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

A barrel of beef or pork weighs 200 lbs.
A barrel of flour weighs 196 lbs.

### PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.

### BOOKS.

A sheet folded into 2 leaves is called a folio.
" " " " 4 " " " " quarto.
" " " " 8 " " " " octavo.
" " " " 12 " " " " duodecimo or 12 mo.
" " " " 18 " " " " 18 mo.

---

* In Leap year February has 29 days.

**Q.** 1. What must be added to 16s. to give a sum equal to £1? What must be added to 13s.? to 18s.? to 7s.? to 19s.? to 11s.? to 10s.?

2. What must be added to 9d. to give a sum equal to 1 shilling? What must be added to 3d.? to 11d.? to 8d.? to 4d.? to 7d.?

3. What must be added to 12 gr. to give a sum equal to 1 dwt.? What must be added to 11 gr.? to 17 gr.? to 23 gr.? to 13 gr.? to 15 gr.? to 5 gr.? to 19 gr.? to 9 gr.?

4. What must be added to 7 Avoirdupois ounces to give a sum equal to 1 lb.? What must be added to 11 oz.? to 9 oz.? to 12 oz.? to 2 oz.?

5. What must be added to 17 lb. to give a sum equal to 1 qr.? What must be added to 19 lb.? to 9 lb.? to 11 lb.? to 23 lb.? to 21 lb.? to 17 lb.? to 15 lb.? to 5 lb.? 25 lb.? to 18 lb.? to 3 lb.?

6. What must be added to 121 sq. in. to give a sum equal to 1 sq. ft.? What must be added to 134 sq. in.? to 141 sq. in.? to 107 sq. in.? to 96 sq. in.? to 87 sq. in.? to 71 sq. in.? to 115 sq. in.? to 111 sq. in.? to 47 sq. in.? to 138 sq. in.? to 104 sq. in.? to 109 sq. in.?

7. How many more than cwt. enough for 1 ton are there in 36 cwt.? in 25 cwt.? in 39 cwt.? in 31 cwt.? in 27 cwt.? in 36 cwt.?

Then how many tons and cwt. are equal in value to 37 cwt.? to 25 cwt.? to 39 cwt.? to 31 cwt.? to 27 cwt.? to 36 cwt.?

8. How many more than enough ℥ for one ℔ are there in 16 ℥? Then how many ℔ and ℥ are there in 16 ℥? in 18 ℥? in 23 ℥? in 20 ℥? in 15 ℥? in 19 ℥? in 18 ℥?

9. What is the value of each of the following, expressed in as high denominations as possible? 181 sq. in.? 39 gr. Troy? 37 shillings? 15 ʒ? 55 lb. Avoirdupois? 21 in.? 74 sq. rd.? 16 sq. ft.? 11 days?

10. What will be the value of 211 cu. ft. expressed in cords and feet? of 36 dwt. in oz. and dwt.? of 107 sec. in minutes and seconds?

**R.** 1. What is the value of 23 lb. + 21 lb. expressed in the highest possible denominations? *Solution.* 23 lb. + 21 lb. = 44 lb. = 1 qr. 16 lb.

2. What is the value of each of the following, expressed in the highest possible denominations?

3. Of 26 lb. + 25 lb.? 16s. + 18s.? 13s. + 19s.?

4. Of 17 hours + 21 h.? 22 h. + 23 h.?
5. Of 22 gr. + 19 gr. Troy weight? 18 dwt. + 16 dwt.?
6. Of 11 oz. + 9 oz. Troy? 10d. + 8d.? 16s. + 18s.?
7. Of 15 dr. + 13 dr.? 9 oz. + 8 oz. Avoirdupois?
8. Of 12 oz. + 15 oz. av.? 26 lbs. + 8 lbs.?
9. Of 11s. + 12s.? 5 qt. + 7 qt. Dry Measure?
10. Of 135 sq. in. + 121 sq. in.? 85 sq. in. + 79 sq. in.?
11. Of 11d. + 8d.?

Much labor can be saved in performing examples like those under this letter, by applying the principles illustrated in the following example:

What is the value of 23 lb. + 21 lb.?

Observing that 23 lb. lack but 5 lb. of being 1 qr., we take 5 lb. from 21 lb. and add it to 23 lb., which gives the following solution: 23 lb. + 21 lb. = 28 lb. + 16 lb. = 1 qr., 16 lb. *Ans.*

Perform all the examples under this letter by the method last explained.

**S.** 1. What is the value of 23 lb. + 26 lb. + 27 lb.?

*Solution.* 23 lb. + 26 lb. = 49 lb. or 1 qr. 21 lb., and 1 qr. 21 lb. + 27 lb. = 1 qr. 48 lb. = 1 qr. + 1 qr. 20 lb. = 2 qr. 20 lb., *Ans.* Or, adding and reducing by naming only results, we say, 23 lb.; 49 lb., or 1 qr. 21 lb.; 1 qr. 48 lb., or 2 qr. 20 lb., *Ans.*

2. What is the value of 24 lb. + 19 lb. + 25 lb.?
3. 23 lb. + 21 lb. + 24 lb.? 17 cwt. + 18 cwt. + 19 cwt.?
4. 13 oz. + 15 oz. + 11 oz. Avoirdupois?
5. 15 dr. + 10 dr. + 13 dr. Av.? 8 ℥ + 11 ℥ + 9 ℥?
6. 18 gr. + 15 gr. + 17 gr. Apothecaries?
7. 7 ʒ + 6 ʒ + 5 ʒ + 7 ʒ?
8. 8 in. + 11 in. + 7 in. + 4 in.?
9. 36 rd. + 29 rd. + 38 rd.? 17s. + 11s. + 16s. + 18s.?
10. 8d. + 6d. + 5d. + 9d.? 47 sec. + 52 sec. + 39 sec.?
11. 13 h. + 21 h. + 17 h. + 22 h.?
12. 132 sq. in. + 100 sq. in. + 140 sq. in.?
13. 126 sq. in. + 134 sq. in. + 127 sq. in.?
14. What is the value of 23 lb. + 26 lb. + 27 lb.?

*Solution.* Since 26 lb. lack but 2 lb. of being 28 lb., or

1 qr., we have 23 lb. + 26 lb. = 1 qr. 21 lb.; and, since 27 lb. lack but 1 lb. of being 1 qr., we have 1 qr. 21 lb. + 27 lb. = 2 qr. 20 lb. Therefore, 23 lb. + 26 lb. + 27 lb. = 2 qr. 20 lb. Or, adding and reducing by naming only results, we say 23 lb.; 1 qr. 21 lb.; 2 qr. 20 lb., *Ans.*

15. Apply the principles illustrated in the last solution to each of the examples under this letter.

NOTE. The pupil should *master* all the forms of solution given above, so that in practice he may use one or the other, as may seem best adapted to the numbers he has to add.

16. What is the value of 7 oz., 11 dr. + 3 oz., 14 dr.?

*Solution.* 7 oz. 11 dr. + 3 oz. 14 dr. = 7 oz. 11 dr. + 3 oz. + 14 dr. = 10 oz. 11 dr. + 14 dr. = 11 oz. 9 dr.

*Solution*, naming only results, 7 oz. 11. dr.; 10 oz. 11 dr.; 11 oz. 9 dr., *Ans.*

17. What is the value of 5 oz. 13 dr. + 4 oz. 5 dr.?

18. 1 qr. 25 lb. + 1 qr. 23 lb.? 12 lb. 13 oz. + 8 lb. 14 oz.?

19. 15 dwt. 21 gr. + 2 dwt. 19 gr.?

20. 8 dwt. 14 gr. + 3 dwt. 13 gr.?

21. 3 oz. 17 dwt. + 2 oz. 8 dwt.? 4 ft. 11 in. + 2 ft. 8 in.?

22. 3 fur. 28 rd. + 2 fur. 27 rd.? 7£ 15s. + 8£ 14s.?

23. 9£ 17s. + 6£ 18s.? 9s. 11d. + 8s. 8d.?

24. 3 da. 19 h. + 2 da. 21 h.?

25. 2 sq. ft. 107 sq. in. + 4 sq. ft. 124 sq. in.?

26. 3 sq. ft. 87 sq. in. + 3 sq. ft. 78 sq. in.?

**T.** 1. What is the value of 4£ 16s. — 3s.?

*Ans.* 4£ 13s.

2. What is the value of 28£ 19s. — 6s.? 17s. 9d. — 7d.?

3. 9£ 18s. — 15s.? 2 qr. 23 lb. — 14 lb.?

4. 6 da. 13 h. — 7 h.? 3 ℥ 7 ʒ — 3 ʒ?

5. 2 ℈ 14 gr. — 12 gr.? 5 sq. ft. 127 sq. in. — 54 sq. in.?

6. What is the value of 28£ — 5s.?

*Solution.* 28£ — 5s. = 27£ 20s. — 5s. = 27£ 15s.

7. What is the value of 19£ — 16s.? 16s. — 7d.?

8. 3 R. — 15 sq. rd.? 2 qr. — 14 lb.? 3 qr. — 23 lb.?

9. 24 lb. — 11 oz. (Avoirdupois)? 8 oz. — 8 dr.?
10. 3 dwt. — 3 gr.? 17 dwt. — 8 gr.? 48 m. — 17 sec.?
11. What is the value of 1 dwt. 14 gr. — 21 gr.?

*1st Solution.* 1 dwt. 14 gr. — 21 gr. = 38 gr. — 21 gr. = 17 gr.

*2d Solution.* Since we cannot subtract 21 gr. from 14 gr., we will subtract it from 1 dwt., or 24 gr., leaving 3 gr., which added to 14 gr. = 17 gr.

*3d Solution.* Subtracting 14 of the 21 grains from the 14 grains, we leave 7 gr. to subtract, which taken from the 24 gr. leaves 17 gr.

NOTE. The second or third forms will usually be more convenient than the first.

12. What is the value of 1 dwt. 21 gr. — 23 gr.?
13. 1 dwt. 7 gr. — 15 gr.? 1 qr. 17 lb. — 19 lb.?
14. 1 qr. 15 lb. — 27 lb.? 1£ 4s. — 17s.?
15. 1s. 1d. — 11d.? 1 m. 27 sec. — 54 sec.?
16. 1 m. 4 sec. — 37 sec.?
17. 1 sq. ft. 123 sq. in. — 138 sq. in.?
18. 1 sq. ft. 14 sq. in. — 103 sq. in.?
19. 1 R. 13 sq. rd. — 29 sq. rd.?
20. What is the value of 4 fur. 21 rd. — 27 rd.?

*Solution.* 4 fur. 21 rd. — 27 rd. = 4 fur. — 6 rd. = 3 fur. 34 rd.

21. What is the value of 7 fur. 18 rd. — 23 rd.?
22. 5 fur. 1 rd. — 37 rd.? 7s. 3d. — 11d.?
23. 9 lb. 4 ℥ — 9 ℥? 2 ℈ 4 gr. — 16 gr.?
24. 4 da. 15 h. — 19 h.? 28 m. 28 sec. — 53 sec.?
25. 17 T. 17 cwt. — 13 cwt. 16 dwt. 4 gr. — 18 gr.?
26. 8 cu. yd. 3 cu. ft. — 23 cu. ft.? 2 m. 1 fur. — 7 fur.?
27. What is the value of 18 gal. 2 qt. — 3 gal. 3 qt.?
28. 17 bu. 1 pk. — 4 bu. 3 pk.? 3 pk. 4 qt. — 2 pk. 7 qt.?
29. 24£ 4s. — 13£ 16s.? 3 qr. 8 lb. — 1 qr. 25 lb.?
30. 3 qr. 16 lb. — 1 qr. 18 lb.? 6 da. 20 h. — 3 da. 22 h.?
31. 4 da. 12 h. — 1 da, 19 h.? 18s. 4d. — 15s. 6d.?
32. 12s. 2d. — 5s. 6d.?

33. 5 sq. yd. 2 sq. ft. — 2 sq. yd. 8 sq. ft.?
34. 86 sq. ft. 14 sq. in. — 75 sq. ft. 121 sq. in.?
35. 97 sq. ft. 111 sq. in. — 85 sq. ft. 143 sq. in.?
36. 49 cu. yd. 14 cu. ft. — 31 cu. yd. 23 cu. ft.?
37. 146 cu. yd. 3 cu. ft. — 29 cu. yd. 26 cu. ft.?
38. 19 yd. 1 qr. — 8 yd. 3 qr.? 44 w. 2 da. — 26 w. 5 da.?
39. 6 da. 4 h. — 4 da. 6 h.? 22 h. 17 m. — 17 h. 22 m.?
40. 47 m. 23 sec. — 23 m. 47 sec.? 5 ʒ 1 ℈ — 2 ʒ 2 ℈?

## SECTION VIII.

The following combinations from the "First Steps in Numbers," are inserted here for convenience of review. For practical examples applying them, and more full illustrations, see First Steps, Sections 10, 11, 12, and 13

**A.** 1. How many are 2 × 2? 3 × 2? 4 × 3? 4 × 2?
2. 2 × 4? 3 × 3? 5 × 2? 2 × 5? 6 × 2? 2 × 6?
3. 4 × 3? 3 × 4? 7 × 2? 2 × 7? 5 × 3? 3 × 5?
4. 4 × 4? 8 × 2? 2 × 8? 9 × 2? 2 × 9? 6 × 3?
5. 3 × 6? 10 × 2? 2 × 10? 4 × 5? 5 × 4?
6. 2 × 2 × 2? 2 × 3 × 2? 3 × 3 × 2?
7. 2 × 2 × 2 × 2? 2 × 2 × 4? 2 × 3 × 3?
8. 2 × 2 × 5? 2 × 2 × 3? 5 × 2 × 2? 4 × 2 × 2?

**B.** 1. How many times 2 = 6? = 18? 20? 4? 12? 16? 8? 14? 10?
2. How many times 3 = 12? 18? 6? 15? 9?
3. How many times 4 = 20? 4? 12? 16? 8?
4. How many times 5 = 10? 20? 5? 15?
5. How many times 6 = 18? 6? 12?
6. How many times 7 = 14? 7?
7. How many times 8 = 16? 8?
8. How many times 9 = 9? 18?
9. How many times 10 = 20? 10?

**C.** 1. 6 = 3 times what number? 9 = 3 times what?
2. 12 = 2 times what? 20 = 5 times what?
3. 15 = 5 times what? 14 = 2 times what?
4. 18 = 9 times what? 8 = 2 times what?
5. 12 = 4 times what? 18 = 6 times what?
6. 15 = 3 times what? 6 = 2 times what?
7. 20 = 10 times what? 14 = 7 times what?
8. 20 = 2 times what? 18 = 3 times what?
9. 8 = 8 times what? 18 = 2 times what?
10. 16 = 4 times what? 20 = 4 times what?
11. 4 = 2 times what? 8 = 4 times what?
12. 12 = 3 times what? 16 = 8 times what?
13. 7 = 7 times what? 12 = 6 times what?
14. 10 = 5 times what? 3 = 3 times what?
15. 10 = 2 times what?

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## SECTION IX.

NOTE. Many of the principles illustrated in this section are more fully illustrated in the First Steps. They are inserted here merely for convenience of review.

**A.** 1. *Explanation.* When any thing or number is divided into two equal parts, the parts are called halves of the thing or number. One of the parts is called one half, and two of the parts are called two halves of the thing or number.

2. *Explanation.* When any thing or number is divided into three equal parts, the parts are called thirds of the thing or number. One of the parts is called one third, two of the parts two thirds, and three of the parts three thirds of the thing or number.

3. *Explanation.* When any thing or number is divided into four equal parts, the parts are called fourths of the thing or number. One of the parts is called one fourth, two of the parts two fourths, three of the parts three fourths, &c.

4. *Explanation.* In like manner, when any thing or number is divided into five equal parts, the parts are called fifths of the thing or number; when divided into six equal parts, the parts are called sixths; when divided into seven equal parts, they are called sevenths; when into eight equal parts, they are called eighths, &c., &c.

5. What are fifths of a thing?

*Ans.* Fifths of a thing are parts obtained by dividing a thing into five equal parts.

6. What are ninths of a thing? fourths? elevenths? tenths? halves? sevenths? twentieths? eighths? thirds? fourteenths? nineteenths?

7. *Explanation.* Parts obtained by dividing a thing into equal parts, are called fractional parts of the thing divided, and the arithmetical expressions for them are called fractions. The figures, expressing the number of parts into which the thing is supposed to be divided, are called the denominator of the fraction, because they show the size or denomination of each part, while those expressing the number of parts we wish to consider, are called the numerator of the fraction.

8. *Explanation.* Fractions are usually expressed by writing the numerator over the denominator, and drawing a line between them.

Thus, five sixths is written $\frac{5}{6}$. 5 is the numerator, and 6 is the denominator.

9. How will the fraction one fourth be expressed by figures?

10. How will 3 fifths be expressed? 7 eighths? 5 sixteenths? 9 tenths? 8 elevenths? 7 twenty-ninths? 14 seventeenths? 63 eighty-fourths? 21 thirty-sevenths?

11. What do you understand by $\frac{5}{8}$ (five eighths) of a thing?

*Ans.* 5 such parts as would be obtained by dividing a thing into eight equal parts.

12. What do you understand by $\frac{5}{6}$ (five sixths) of a thing? by $\frac{9}{10}$ (nine tenths)? by $\frac{21}{33}$ (twenty-one thirty-thirds)? by $\frac{4}{5}$? $\frac{18}{19}$? $\frac{7}{11}$? $\frac{3}{8}$? $\frac{37}{43}$? $\frac{56}{81}$? $\frac{6}{5}$? $\frac{3}{7}$? $\frac{8}{3}$? $\frac{5}{17}$? $\frac{20}{49}$? $\frac{23}{100}$? $\frac{273}{467}$? $\frac{471}{542}$? $\frac{323}{450}$? $\frac{235}{400}$? $\frac{117}{200}$? $\frac{675}{851}$? $\frac{851}{675}$? $\frac{13}{14}$? $\frac{14}{13}$? $\frac{144}{128}$? $\frac{365}{157}$?

13. What does the denominator and what does the numerator in each of the above fractions show?

*Ans.* In the first, $\frac{5}{6}$, the denominator 6 shows the parts are such as would be obtained by dividing the thing considered into six equal parts, and the numerator 5 shows that five such parts are taken. In the second, $\frac{9}{10}$, the denominator 10 shows, &c.

14. How many halves of a thing make the whole thing?

*Ans.* 2; because halves of a thing are obtained by dividing the thing into two equal parts.

15. How many thirds of a thing make the whole thing? How many sixths? ninths? twelfths? seventeenths? thirty-eighths? sixteenths? two hundredths? forty-fifths?

**B.** Let the pupil prove the truth of the following propositions:

*Prop.* 1. When the numerator of a fraction is less than its denominator, the value of the fraction is less than 1.

*Prop.* 2. When the numerator of a fraction is equal to its denominator, the fraction is equal to 1.

*Prop.* 3. When the numerator of a fraction is greater than its denominator, the value of the fraction is greater than 1.

1. A fraction representing a value less than 1, is called a *proper fraction*; as, $\frac{4}{5}$, $\frac{7}{11}$.

2. A fraction representing a value equal to, or greater than, 1, is called an *improper fraction*; as, $\frac{8}{8}$, $\frac{12}{9}$, $\frac{23}{17}$.

3. How will the numerator of a proper fraction compare with its denominator?

4. How will the numerator of an improper fraction compare with its denominator?

5. Which of the fractions under number 12, letter A, are proper, and which improper?

**C.** 1. $\frac{3}{5} = \frac{1}{5}$ of what?

*Ans.* $\frac{3}{5}$, or $\frac{3}{5}$ of $1 = \frac{1}{5}$ of 3; because, if 3 equal things were divided into 5 equal parts, each part would be equivalent to 3 such parts as would be obtained by dividing 1 thing into 5 equal parts.

2. $\frac{4}{7} = \frac{1}{7}$ of what? $\frac{8}{10} = \frac{1}{10}$ of what? $\frac{5}{11} = \frac{1}{11}$ of what?
3. $\frac{8}{9} = \frac{1}{9}$ of what? $\frac{2}{3} = \frac{1}{3}$ of what? $\frac{15}{16} = \frac{1}{16}$ of what?
4. $\frac{8}{11} = \frac{1}{11}$ of what? $\frac{4}{9} = \frac{1}{9}$ of what?
5. $\frac{3}{10} = \frac{1}{10}$ of what? $\frac{6}{7} = \frac{1}{7}$ of what? $\frac{3}{4} = \frac{1}{4}$ of what?
6. $\frac{4}{5} = \frac{1}{5}$ of what? $\frac{5}{6} = \frac{1}{6}$ of what? $\frac{7}{8} = \frac{1}{8}$ of what?
7. $\frac{9}{10} = \frac{1}{10}$ of what?
8. To what is $\frac{1}{7}$ of 4 equal?

*Ans.* $\frac{1}{7}$ of 4 is equal to $\frac{4}{7}$ of 1.

9. To what is $\frac{1}{5}$ of 2 equal? $\frac{1}{8}$ of 3? $\frac{1}{6}$ of 5? $\frac{1}{9}$ of 7?
10. $\frac{1}{3}$ of 2? $\frac{1}{8}$ of 7? $\frac{1}{4}$ of 3? $\frac{1}{9}$ of 5? $\frac{1}{8}$ of 5? $\frac{1}{7}$ of 6?
11. $\frac{1}{10}$ of 7? $\frac{1}{5}$ of 3? $\frac{1}{12}$ of 11? $\frac{1}{13}$ of 9? $\frac{1}{17}$ of 6?
12. $\frac{1}{12}$ of 11? $\frac{1}{11}$ of 6? $\frac{1}{10}$ of 3? $\frac{1}{13}$ of 8?

**D.** 1. $1 =$ how many more than $\frac{3}{11}$?

*Solution.* $1 = \frac{11}{11}$, and $\frac{11}{11} = \frac{8}{11}$ more than $\frac{3}{11}$. Therefore, $1 = \frac{8}{11}$ more than $\frac{3}{11}$.

2. $1 =$ how many more than $\frac{2}{9}$? than $\frac{1}{7}$? than $\frac{5}{13}$? than $\frac{4}{7}$?

3. 1 = how many more than $\frac{8}{15}$? than $\frac{8}{21}$? than $\frac{1}{6}$? than $\frac{1}{144}$?

4. 1 = how many more than $\frac{8}{71}$? than $\frac{4}{17}$? than $\frac{1}{100}$? than $\frac{7}{87}$?

5. 1 = how many more than $\frac{29}{100}$? than $\frac{18}{37}$? than $\frac{4}{43}$? than $\frac{16}{271}$?

6. 1 = how many more than $\frac{13}{48}$? than $\frac{5}{19}$? than $\frac{6}{7}$? than $\frac{8}{65}$?

7. 1 = how many more than $\frac{14}{37}$? than $\frac{1}{1000}$? than $\frac{34}{227}$? than $\frac{132}{323}$?

8. How many more than 1 are $\frac{11}{9}$?

*Solution.* Since $\frac{11}{9} = \frac{2}{9}$ more than $\frac{9}{9}$, it must equal $\frac{2}{9}$ more than 1.

9. How many more than 1 are $\frac{8}{5}$? $\frac{7}{4}$? $\frac{12}{7}$? $\frac{9}{7}$? $\frac{20}{13}$?

10. How many more than 1 are $\frac{15}{11}$? $\frac{17}{14}$? $\frac{3}{2}$? $\frac{5}{3}$? $\frac{30}{22}$?

11. How many more than 1 are $\frac{30}{19}$? $\frac{30}{17}$? $\frac{41}{23}$? $\frac{9}{5}$? $\frac{17}{9}$?

12. How many more than 1 are $\frac{100}{99}$? $\frac{147}{99}$? $\frac{231}{148}$? $\frac{475}{308}$?

13. How many more than 1 are $\frac{25}{14}$? $\frac{20}{11}$? $\frac{1000}{547}$?

NOTE. A whole number and a fraction considered together form what is called a mixed number, as $1\frac{4}{5}$, $2\frac{7}{8}$, $45\frac{19}{27}$, &c.

14. To what mixed number is each of the improper fractions under numbers 9, 10, 11, 12, and 13, equal?

*Ans.* $\frac{8}{5} = 1\frac{3}{5}$, $\frac{7}{4} = 1\frac{3}{4}$, &c.

15. To what mixed number is $\frac{9}{7}$ equal? $\frac{9}{6}$? $\frac{11}{10}$? $\frac{12}{9}$? $\frac{11}{6}$? $\frac{13}{9}$?

16. To what mixed number is $\frac{45}{37}$ equal? $\frac{53}{38}$? $\frac{149}{87}$? $\frac{47}{27}$? $\frac{21}{19}$?

**E.** 1. What is the value of $\frac{9}{11} + \frac{7}{11}$?

*Solution.* $\frac{9}{11} + \frac{7}{11} = \frac{16}{11} = 1\frac{5}{11}$, *Ans.*

2. What is the value of $\frac{7}{9} + \frac{4}{9}$? $\frac{6}{7} + \frac{5}{7}$? $\frac{7}{10} + \frac{9}{10}$?

3. What is the value of $\frac{9}{13} + \frac{11}{13}$? $\frac{6}{17} + \frac{16}{17}$?

4. What is the value of $\frac{8}{15} + \frac{13}{15}$? $\frac{23}{25} + \frac{19}{25}$?

5. What is the value of $\frac{37}{41} + \frac{29}{41}$? $\frac{87}{100} + \frac{94}{100}$?

6. What is the value of $\frac{19}{27} + \frac{16}{27}$? $\frac{8}{9} + \frac{4}{9}$?

7. What is the value of $\frac{67}{83} + \frac{59}{83}$? $\frac{124}{135} + \frac{129}{135}$?

8. What is the value of $\frac{7}{11} + \frac{9}{11}$?

*Solution.* Since $\frac{9}{11}$ is $\frac{2}{11}$ less than 1, $\frac{9}{11} + \frac{7}{11}$ must equal $\frac{11}{11} + \frac{5}{11} = 1\frac{5}{11}$.

9. Perform by the method last explained each of the previous examples under this letter.

10. How many are $4 + 3 + \frac{7}{8}$? Then how many are $4 + 3\frac{7}{8}$? $4\frac{7}{8} + 3$?

11. How many are $6 + 5 + \frac{17}{23}$? Then how many are $6 + 5\frac{17}{23}$? $5\frac{17}{23} + 6$? $6\frac{17}{23} + 5$?

12. How many are $8 + 7\frac{3}{5}$? $6 + 5\frac{4}{9}$? $12 + 21\frac{3}{8}$? $41 + 4\frac{7}{8}$? $63\frac{4}{7} + 18$?

13. How many are $98\frac{3}{11} + 41$? $37 + 16\frac{7}{8}$? $43 + 5\frac{17}{18}$? $85\frac{2}{3} + 16$?

14. How many are $\frac{9}{11} + \frac{6}{11} + 7$? Then how many are $7 + \frac{9}{11} + \frac{6}{11}$? $7\frac{9}{11} + \frac{6}{11}$? $\frac{9}{11} + 7\frac{6}{11}$?

15. How many are $\frac{47}{51} + \frac{38}{51} + 69$? Then how many are $69 + \frac{47}{51} + \frac{38}{51}$? $69\frac{47}{51} + \frac{38}{51}$? $\frac{47}{51} + 69\frac{38}{51}$?

16. How many are $3\frac{7}{8} + \frac{5}{8}$? $4\frac{9}{10} + \frac{7}{10}$? $8\frac{12}{14} + \frac{11}{14}$? $5\frac{3}{7} + \frac{6}{7}$?

17. How many are $\frac{8}{9} + 8\frac{8}{9}$? $\frac{5}{7} + 3\frac{6}{7}$? $18\frac{29}{48} + \frac{23}{48}$? $14\frac{7}{11} + \frac{9}{11}$?

18. How many are $4\frac{4}{5} + 3 + \frac{3}{5}$? Then how many are $4\frac{4}{5} + 3\frac{3}{5}$?

19. How many are $3\frac{5}{7} + 8\frac{6}{7}$? $9\frac{7}{11} + 8\frac{9}{11}$? $5\frac{8}{9} + 8\frac{5}{9}$? $6\frac{15}{17} + 15\frac{6}{17}$?

20. How many are $18\frac{4}{11} + 4\frac{9}{11}$? $3\frac{29}{33} + 4\frac{31}{33}$? $8\frac{186}{207} + 3\frac{184}{207}$?

21. How many are $\frac{14}{17} + \frac{13}{17} + \frac{12}{17}$?

*Solution.* $\frac{14}{17} + \frac{13}{17} = \frac{27}{17}$ or $1\frac{10}{17}$, and $1\frac{10}{17} + \frac{12}{17} = 1\frac{22}{17} = 2\frac{5}{17}$. *Ans.* Or, naming only results, we should say, $\frac{14}{17}$, $\frac{27}{17}$, or $1\frac{10}{17}$, $1\frac{22}{17}$, $2\frac{5}{17}$, *Ans.*

*2nd Solution.* Since $\frac{14}{17}$ is $\frac{3}{17}$ less than 1, $\frac{14}{17} + \frac{13}{17} = \frac{17}{17} + \frac{10}{17} = 1\frac{10}{17}$. Now since $\frac{10}{17}$ is $\frac{7}{17}$ less than 1, $1\frac{10}{17} + \frac{12}{17}$ must equal $1\frac{17}{17} + \frac{5}{17} = 2\frac{5}{17}$. Or, naming only results, we should say, $\frac{14}{17}$, $1\frac{10}{17}$, $2\frac{5}{17}$, *Ans.*

22. How many are $\frac{8}{9} + \frac{7}{9} + \frac{5}{9}$? $\frac{3}{7} + \frac{6}{7} + \frac{5}{7}$? $\frac{12}{13} + \frac{10}{13} + \frac{8}{13}$?

23. How many are $\frac{16}{23} + \frac{22}{23} + \frac{19}{23}$? $\frac{9}{14} + \frac{13}{14} + \frac{11}{14}$? $\frac{24}{27} + \frac{21}{27} + \frac{15}{27} + \frac{26}{27}$?

24. How many are $\frac{18}{21} + \frac{13}{21} + \frac{16}{21} + \frac{20}{21}$? $\frac{3}{10} + \frac{2}{10} + \frac{8}{10} + \frac{6}{10}$?

25. How many are $3\frac{2}{3} + 4\frac{2}{3} + 5\frac{1}{3} + 7\frac{2}{3}$? $9\frac{3}{7} + 4\frac{6}{7} + 5\frac{5}{7} + 8\frac{6}{7}$?

26. How many are $6\frac{15}{18} + 2\frac{9}{18} + 4\frac{17}{18} + \frac{5}{18} + 3\frac{13}{18} + 2\frac{17}{18} + \frac{11}{18}$?

27. How many are $3\frac{14}{15} + 6\frac{12}{15} + 2\frac{11}{15} + 13\frac{13}{15} + \frac{7}{15} + \frac{9}{15} + 6\frac{9}{15} + 7\frac{14}{15}$?

28. How many are $8\frac{94}{99} + 2\frac{81}{99} + 3\frac{89}{99} + 6\frac{70}{99} + 3\frac{89}{99} + \frac{76}{99} + \frac{23}{99} + 1\frac{98}{99}$?

29. How many are $3\frac{68}{75} + 2\frac{78}{75} + 4\frac{8}{75} + 2\frac{74}{75} + \frac{73}{75} + 1\frac{71}{75} + \frac{67}{75}$?

**F.** 1. How many are $1 - \frac{2}{5}$?

*Solution.* $1 - \frac{2}{5} = \frac{5}{5} - \frac{2}{5} = \frac{3}{5}$, *Ans.*

2. How many are $1 - \frac{4}{7}$? $1 - \frac{5}{9}$? $1 - \frac{16}{37}$? $1 - \frac{3}{32}$? $1 - \frac{7}{11}$?

3. How many are $1 - \frac{13}{59}$? $1 - \frac{48}{73}$? $1 - \frac{66}{125}$? $1 - \frac{13}{144}$? $1 - \frac{19}{86}$?

4. How many are $1\frac{7}{24} - \frac{19}{24}$?

*1st Solution.* $1\frac{7}{24} - \frac{19}{24} = \frac{31}{24} - \frac{19}{24} = \frac{12}{24}$, *Ans.*

*2nd Solution.* $1\frac{7}{24} - \frac{19}{24} = 1 - \frac{19}{24} + \frac{7}{24} = \frac{5}{24} + \frac{7}{24} = \frac{12}{24}$, *Ans.*

*3d Solution.* $1\frac{7}{24} - \frac{19}{24} = 1\frac{7}{24} - \frac{7}{24} - \frac{12}{24} = 1 - \frac{12}{24} = \frac{12}{24}$, *Ans.*

5. How many are $1\frac{5}{9} - \frac{7}{9}$? $1\frac{18}{37} - \frac{35}{37}$? $1\frac{14}{36} - \frac{16}{36}$? $1\frac{7}{83} - \frac{12}{83}$?

6. How many are $1\frac{2}{11} - \frac{8}{11}$? $1\frac{137}{189} - \frac{148}{179}$? $1\frac{15}{67} - \frac{28}{67}$?

7. How many are $7 - \frac{13}{25}$?

*Solution.* $7 = 6\frac{25}{25}$, and $6\frac{25}{25} - \frac{13}{25} = 6\frac{12}{25}$. Therefore, $7 - \frac{13}{25} = 6\frac{12}{25}$, *Ans.*

8. How many are $8 - \frac{14}{27}$? $10 - \frac{3}{4}$? $28 - \frac{1}{2}$? $37 - \frac{1}{8}$? $41 - \frac{8}{41}$?

9. How many are $12 - \frac{14}{15}$? $87 - \frac{224}{237}$? $49 - \frac{3}{8}$? $5 - \frac{461}{473}$?

10. How many are $16\frac{5}{19} - \frac{7}{9}$?

*Solution.* $16\frac{5}{9} - \frac{7}{9} = 15\frac{14}{9} - \frac{7}{9} = 15\frac{7}{9}$, *Ans.*

*2nd Solution.* $16\frac{5}{9} - \frac{7}{9} = 16 - \frac{7}{9} + \frac{5}{9} = 15\frac{2}{9} + \frac{5}{9} = 15\frac{7}{9}$, *Ans.*

*3d Solution.* $16\frac{5}{9} - \frac{7}{9} = 16\frac{5}{9} - \frac{5}{9} - \frac{2}{9} = 15\frac{7}{9}$, *Ans.*

11. How many are $17\frac{3}{10} - \frac{9}{10}$? $14\frac{2}{7} - \frac{6}{7}$? $3\frac{14}{25} - \frac{16}{25}$?

12. How many are $15\frac{4}{43} - \frac{28}{43}$? $9\frac{18}{79} - \frac{27}{79}$? $4\frac{217}{456} - \frac{220}{456}$?

13. How many are $3\frac{11}{13} - \frac{12}{13}$? $7\frac{9}{35} - \frac{24}{35}$? $6\frac{411}{537} - \frac{527}{537}$?

14. How many are $4\frac{7}{8} - 2 - \frac{5}{8}$? $4\frac{7}{8} - 2\frac{5}{8}$?

15. How many are $12\frac{4}{13} - 3 - \frac{9}{13}$? $12\frac{4}{13} - 3\frac{9}{13}$?

16. How many are $9\frac{7}{19} - 2\frac{13}{19}$? $8\frac{16}{25} - 3\frac{22}{25}$? $17\frac{4}{83} - 5\frac{58}{83}$?

17. How many are $25\frac{4}{13} - 18\frac{12}{13}$? $44\frac{9}{44} - 22\frac{25}{44}$? $37\frac{19}{43} - 19\frac{37}{43}$?

18. How many are $49\frac{31}{65} - 31\frac{48}{65}$? $136\frac{8}{21} - 43\frac{15}{21} - 25\frac{19}{21}$?

19. How many are $64\frac{5}{7} - 31\frac{2}{7} - 28\frac{6}{7}$? $37\frac{1}{2} + 6\frac{1}{2} - 27\frac{1}{2}$?

20. How many are $44\frac{2}{5} + 27\frac{4}{5} - 23\frac{3}{5}$? $16\frac{2}{3} + 28\frac{2}{3} - 19\frac{2}{3} - 13\frac{1}{3}$?

21. How many are $57\frac{4}{7} + 25\frac{5}{7} - 16\frac{6}{7} - 23\frac{5}{7}$? $64\frac{5}{19} - 17\frac{8}{19}$?

22. How many are $44\frac{2}{9} - 27\frac{8}{9} + 23\frac{5}{9} - 16\frac{1}{9} - 5\frac{4}{9}$?

23. How many are $28\frac{31}{33} - 6\frac{23}{33} + 5\frac{8}{33} - 2\frac{32}{33}$?

24. How many are $27\frac{3}{8} - 19\frac{5}{8} - 2\frac{3}{8} + 6\frac{7}{8} + 4\frac{3}{8}$?

25. How many are $100\frac{7}{16} - 16\frac{15}{16} - 14\frac{9}{16} - 18\frac{15}{16} - 21\frac{3}{16}$?

**G.** 1. Julia's father gave her $\frac{18}{50}$ of a dollar, her mother gave her $\frac{13}{50}$, her uncle gave her $\frac{11}{50}$, and her brother gave her enough to make up 1 dollar. What part of a dollar did her brother give her? She spent $\frac{9}{25}$ of a dollar for books, $\frac{8}{25}$ for pens and pencils, $\frac{2}{25}$ for candy, and gave the rest to a charitable society. How much did she give to the charitable society?

2. $\frac{5}{28}$ of the scholars in a certain school were more than 15 years of age, $\frac{21}{28}$ were between 5 and 15, and the rest were under 5. What part of them were under 5? $\frac{20}{39}$ of them were girls, and the rest were boys. What part of the whole were boys? $\frac{9}{37}$ of them read in the first class, $\frac{12}{37}$ in the second, $\frac{7}{37}$ in the third, and the rest read in the primer. What part of the whole read in the primer?

3. A farmer has $17\frac{9}{14}$ acres of tillage, $27\frac{11}{14}$ acres of pasturage, $21\frac{5}{14}$ acres of woodland, and $7\frac{9}{14}$ acres of orchard. How many acres of land has he in all? If he should buy $12\frac{13}{14}$ acres of meadow, and $9\frac{3}{14}$ acres of salt-marsh, how many acres of land would he have?

4. Jabez had 4 pieces of fish-line. The first piece contained $6\frac{4}{5}$ yds., the second $2\frac{3}{5}$ yds., the third $3\frac{2}{5}$ yds., and the fourth $9\frac{1}{5}$ yds. How many yards of line has he in all? How many more has he in all than there are in the first piece? than in the second? than in the third? than in the fourth? If he give the first piece, and $2\frac{4}{5}$ yards from the third, to Abner, how many yards will he have left?

5. Alfred spent $1\frac{4}{11}$ hours per day in studying arithmetic, $2\frac{1}{11}$ hours in studying his reading and spelling lessons, $1\frac{9}{11}$ hours in studying his geography, and $\frac{10}{11}$ of an hour in studying his grammar. How many hours did he spend in studying all his lessons? How many more hours in studying his reading and spelling lessons, than his arithmetic? than his geography? than his grammar? Besides the time spent in studying, he spent 2 hours per day in reciting, $1\frac{9}{11}$ hours in doing errands and working for his father, $2\frac{4}{11}$ hours in playing, $1\frac{7}{11}$ hours at his meals, and the rest of the time in sleeping. How many hours of each day did he spend in sleeping? How many more hours did he spend in sleeping than in studying? than in playing? than in doing errands? than in reciting? than at his meals?

6. Farmer Bowen had $17\frac{4}{23}$ tons of hay in his barn, and $15\frac{7}{23}$ tons in his stacks. How many tons did he have in all? He moved $3\frac{15}{23}$ tons from his stacks into his barn. How many tons did he then have in his barn? in his stacks? How many more tons in his barn than in his stacks?

7. A girl picked 26 quarts of blackberries in a week. She picked $3\frac{5}{12}$ qts. on Monday, $5\frac{1}{12}$ qts. on Tuesday, $4\frac{7}{12}$ qts. on Wednesday, $3\frac{11}{12}$ qts. on Thursday, $5\frac{5}{12}$ qts. on Friday, and the rest on Saturday forenoon. How many quarts did she pick on Saturday forenoon? She sold $12\frac{8}{9}$ qts., and gave the rest to her mother. How many quarts did she give to her mother? Her mother used $2\frac{4}{9}$ quarts for pies, $1\frac{7}{9}$ quarts for puddings, and then dried the rest. How many quarts did she dry?

8. Farmer Wilson and his son John, were engaged in gathering apples. Farmer Wilson gathered $8\frac{1}{7}$ barrels on Monday, $7\frac{4}{7}$ on Tuesday, $9\frac{5}{7}$ on Wednesday, $6\frac{3}{7}$ on Thursday, $7\frac{2}{7}$ on Friday, and $8\frac{3}{7}$ on Saturday. John gathered $5\frac{2}{7}$ barrels on Monday, $4\frac{6}{7}$ on Tuesday, $6\frac{3}{7}$ on Wednesday, $3\frac{5}{7}$ on Thursday, and on Friday, after having gathered $1\frac{1}{7}$ barrels, he fell from a tree and hurt him, so that he was unable to do any more work during the week. How many did Farmer Wilson gather during the week?

How many did John gather? How many did they both gather? How many more did Farmer Wilson gather than John?

9. The same farmer sold $8\frac{1}{8}$ barrels of his apples to Mr. Allen, $7\frac{2}{8}$ barrels to Mr. Hewitt, $11\frac{7}{8}$ to Mr. Willis, $14\frac{3}{8}$ to Mr. Brown, $12\frac{5}{8}$ to Mr. Jones, $7\frac{6}{8}$ to Mr. Bell, and kept the remainder for his own use? How many barrels did he keep for his own use? How many did he sell? He received from Mr. Allen, in payment for his apples, $15\frac{5}{16}$ dollars, from Mr. Hewitt $13\frac{3}{16}$ dollars, from Mr. Willis $25\frac{7}{16}$ dollars, from Mr. Brown $23\frac{9}{16}$ dollars, from Mr. Jones $22\frac{11}{16}$ dollars, and from Mr. Bell $11\frac{1}{16}$ dollars. How many dollars did he receive in all?

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## SECTION X.

**A.** 1. How many are 7 × 3?

*Solution.* 7 times 3 = one more three than 6 times 3; 6 times 3 = 18, and 18 + 3 = 21. Therefore, 7 times 3 = 21.

2. How many are 3 × 7? 8 × 3? 3 × 8? 6 × 4?
3. 4 × 6? 5 × 5? 9 × 3? 3 × 9? 7 × 4?
4. 4 × 7? 6 × 5? 5 × 6? 10 × 3? 3 × 10?
5. 8 × 4? 4 × 8? 7 × 5? 5 × 7? 6 × 6?
6. 9 × 4? 4 × 9? 10 × 4? 4 × 10? 8 × 5?
7. 5 × 8? 7 × 6? 6 × 7? 9 × 5? 5 × 9?
8. 8 × 6? 6 × 8? 7 × 7? 10 × 5? 5 × 10?
9. 9 × 6? 6 × 9? 7 × 8? 8 × 7? 10 × 6?
10. 6 × 10? 9 × 7? 7 × 9? 8 × 8? 10 × 7?
11. 7 × 10? 9 × 8? 8 × 9? 10 × 8? 8 × 10?
12. 9 × 9? 10 × 9? 9 × 10? 10 × 10?

**B.** 1. How many times 4 = 20? 40? 28? 32? 24? 36?

2. How many times 10 = 40? 80? 20? 50? 90? 30? 60? 100? 70?

3. How many times 7 = 28? 49? 63? 35? 70? 21? 56? 42? 14?

4. How many times 3 = 30? 21? 27? 24?

5. How many times 8 = 40? 64? 24? 32? 80? 48? 72? 56?

6. How many times 5 = 30? 25? 50? 40? 35? 45? 20?

7. How many times 9 = 45? 36? 27? 81? 63? 90? 54? 72?

8. How many times 6 = 36? 48? 30? 24? 42? 54? 60?

**C.** 1. How many are 48 ÷ 6? 28 ÷ 7? 24 ÷ 6?

2. 56 ÷ 7? 70 ÷ 10? 40 ÷ 5? 32 ÷ 8? 90 ÷ 10?

3. 81 ÷ 9? 40 ÷ 8? 30 ÷ 6? 72 ÷ 8? 42 ÷ 7?

4. 64 ÷ 8? 72 ÷ 9? 35 ÷ 5? 25 ÷ 5? 63 ÷ 9?

5. 21 ÷ 7? 56 ÷ 8? 27 ÷ 9? 70 ÷ 7? 21 ÷ 7?

6. 28 ÷ 4? 36 ÷ 4? 45 ÷ 9? 24 ÷ 4? 27 ÷ 3?

7. 45 ÷ 5? 21 ÷ 3? 54 ÷ 6? 36 ÷ 9? 35 ÷ 7?

8. 30 ÷ 3? 63 ÷ 7? 100 ÷ 10? 48 ÷ 8? 32 ÷ 4?

9. 54 ÷ 9? 50 ÷ 10? 36 ÷ 6? 30 ÷ 5? 50 ÷ 5?

10. 80 ÷ 8? 42 ÷ 6?

**D.** 1. Reduce 7 to fourths.

*Solution.* Since 1 = 4 fourths, 7 must equal 7 times 4 fourths, or 28 fourths. Therefore, $7 = \frac{28}{4}$.

2. Reduce 6 to fifths. 9 to thirds. 5 to sevenths.

3. 3 to ninths. 8 to sixths. 4 to sevenths. 3 to eighths.

4. 6 to sevenths. 4 to ninths. 8 to sevenths. 10 to ninths.

5. 9 to eighths. 8 to ninths. 7 to eighths. 6 to eighths.

6. 9 to ninths. 8 to thirds. 4 to eighths. 3 to sevenths.

7. 6 to sixths. 7 to sevenths. 8 to ninths. 10 to fifths.

8. Reduce $4\frac{5}{9}$ to ninths. Since 1 = 9 ninths, 4 must equal 4 times 9 ninths, or 36 ninths; and 5 ninths added are 41 ninths. Therefore, $4\frac{5}{9} = \frac{41}{9}$.

9. Reduce $3\frac{5}{7}$ to sevenths. $4\frac{2}{7}$ to sevenths. $8\frac{1}{6}$ to sixths.

10. $4\frac{7}{10}$ to tenths. $5\frac{4}{5}$ to fifths. $9\frac{1}{3}$ to thirds. $7\frac{5}{6}$ to sixths.

11. $6\frac{4}{9}$ to ninths. $5\frac{3}{8}$ to eighths. $6\frac{7}{8}$ to eighths.

12. $9\frac{7}{9}$ to ninths. $10\frac{2}{3}$ to thirds. $9\frac{5}{6}$ to sixths. $6\frac{5}{6}$ to sixths.

13. $9\frac{6}{7}$ to sevenths. $7\frac{6}{9}$ to ninths. $6\frac{5}{9}$ to ninths.

14. $9\frac{3}{8}$ to eighths. $10\frac{3}{4}$ to fourths. $3\frac{7}{8}$ to eighths.

15. $3\frac{7}{9}$ to ninths. $8\frac{4}{5}$ to fifths. $5\frac{3}{8}$ to eighths. $7\frac{2}{3}$ to thirds

16. $8\frac{3}{9}$ to ninths. $4\frac{5}{7}$ to sevenths. $10\frac{6}{7}$ to sevenths.
17. $9\frac{1}{6}$ to sixths. $7\frac{1}{7}$ to sevenths. $8\frac{8}{9}$ to ninths.
18. $8\frac{3}{4}$ to fourths. $4\frac{8}{9}$ to ninths. $8\frac{5}{9}$ to ninths.
19. $9\frac{1}{4}$ to fourths. $7\frac{4}{9}$ to ninths.

**E.** 1. Reduce $\frac{48}{6}$ to ones.

*Solution.* Since 6 sixths $=$ 1, 48 sixths must equal as many ones as there are times 6 sixths in 48 sixths, which are 8 times. Therefore, $\frac{48}{6} = 8$ ones, or 8.

Reduce each of the following expressions to ones.

2. $\frac{36}{4}$. $\frac{40}{10}$. $\frac{20}{5}$. $\frac{30}{3}$. $\frac{64}{8}$. $\frac{72}{9}$. $\frac{90}{10}$. $\frac{30}{10}$. $\frac{48}{6}$. $\frac{36}{9}$.
3. $\frac{80}{8}$. $\frac{28}{7}$. $\frac{28}{4}$. $\frac{70}{10}$. $\frac{21}{3}$. $\frac{56}{8}$. $\frac{25}{5}$. $\frac{90}{9}$. $\frac{16}{8}$. $\frac{60}{10}$.
4. $\frac{27}{9}$. $\frac{70}{7}$. $\frac{63}{9}$. $\frac{42}{6}$. $\frac{72}{8}$. $\frac{24}{3}$. $\frac{80}{10}$. $\frac{24}{8}$. $\frac{54}{9}$. $\frac{35}{7}$.
5. $\frac{40}{4}$. $\frac{21}{3}$. $\frac{81}{9}$. $\frac{54}{6}$. $\frac{50}{5}$. $\frac{63}{7}$. $\frac{48}{8}$. $\frac{30}{3}$. $\frac{45}{9}$. $\frac{20}{2}$.
6. $\frac{42}{7}$. $\frac{56}{7}$. $\frac{36}{6}$. $\frac{100}{10}$.

**F.** 1. How many are 8 times $\frac{3}{4}$? *Solution.* 8 times $\frac{3}{4} = \frac{24}{4}$, which is equal to 6 ones. Therefore, 8 times $\frac{3}{4} = 6$.

2. How many are $6 \times \frac{2}{3}$? $9 \times \frac{4}{6}$? $8 \times \frac{3}{6}$? $9 \times \frac{2}{3}$?
3. How many are $7 \times \frac{6}{7}$? $9 \times 4\frac{1}{3}$? $9 \times 6\frac{2}{6}$?
4. How many are $8 \times 7\frac{3}{4}$? $9 \times 8\frac{2}{3}$? $3 \times 7\frac{6}{9}$?
5. How many are $6 \times 9\frac{2}{3}$? $4 \times 7\frac{6}{8}$? $10 \times 8\frac{3}{5}$?
6. How many are $9 \times 7\frac{4}{6}$? $8 \times 9\frac{7}{8}$?
7. $6 =$ how many times $\frac{3}{4}$? *Solution.* $6 = \frac{24}{4}$, and $\frac{3}{4}$ are contained in $\frac{24}{4}$, 8 times. Therefore, $6 = 8$ times $\frac{3}{4}$.
8. $7\frac{1}{5} =$ how many times $\frac{4}{5}$? *Solution.* $7\frac{1}{5} = \frac{36}{5}$, and $\frac{4}{5}$ are contained in $\frac{36}{5}$, 9 times. Therefore, $7\frac{1}{5} = 9$ times $\frac{4}{5}$.
9. $4 =$ how many times $\frac{2}{3}$? $5\frac{4}{9} =$ how many times $\frac{7}{9}$?
10. $3\frac{4}{7} =$ how many times $\frac{5}{7}$? $5\frac{1}{4} =$ how many times $\frac{3}{4}$?
11. $8 =$ how many times $\frac{4}{5}$? $10\frac{4}{5} =$ how many times $1\frac{1}{5}$?
12. $9\frac{4}{5} =$ how many times $1\frac{2}{5}$? $7\frac{1}{9} =$ how many times $\frac{8}{9}$?
13. $5\frac{5}{6} =$ how many times $1\frac{1}{6}$? $6\frac{2}{3} =$ how many times $1\frac{2}{3}$?
14. $10 =$ how many times $1\frac{2}{3}$? $9\frac{1}{3} =$ how many times $2\frac{1}{3}$?
15. $9\frac{3}{5} =$ how many times $1\frac{3}{5}$? $7\frac{7}{8} =$ how many times $1\frac{1}{8}$?

**G.** 1. How many are $7 \times 3 + 4, \div 5$?
2. $9 \times 4 - 6, \div 5$? $8 \times 3 + 3, \div 3$?

3. $48 \div 6 - 4, \times 9$? $8 \times 8 + 6, \div 7$?
4. $32 \div 4, \times 6, \div 8$? $10 \times 10 - 28, \div 9$?
5. $7 \times 4 + 2, \div 6$? $9 \times 8 + 9, \div 9$?
6. $5 \times 9 - 9, \div 4$? $7 \times 9 + 1, \div 8$?
7. $3 \times 8 + 24, \div 6$? $7 \times 8 + 4, \div 10$?
8. $7 \times 3 + 6 \times 4$? $8 \times 5 + 6 \times 7$?
9. $9 \times 6 + 40 \div 8$? $10 \times 8 - 8, \div 9$?
10. $7 \times 7 + 3 \times 8$? $6 \times 6 + 4 \times 7$?
11. $6 \times 9 + 7 \times 8$? $7 \times 9 + 18, \div 9$?
12. $10 \times 6 + 8 \times 9$? $7 \times 9 + 8 \times 8$?
13. $6 \times 10 + 9 \times 9$? $6 \times 4 + 3 \times 9 + 4 \times 8$?
14. $9 \times 7 + 8 \times 9 + 7 \times 6$?
15. $8 \times 9 - 3 \times 8 + 7 \times 9$?
16. $5 \times 3 + 6 \times 6 + 7 \times 7$?
17. $9 \times 3 + 4 \times 8 + 7 \times 6$?
18. $9 \times 9 - 5 \times 7 + 7 \times 8$?
19. $8 \times 8 + 6 \times 6, \div 10$? $5 \times 9 + 6 \times 4 - 7 \times 7$?
20. $5 \times 10 - 6 \times 6 + 3 \times 7, \div 5$?
21. $7 \times 5 + 7 \times 6 - 4 \times 8, \div 9$?
22. $8 \times 3 + 4 \times 9 - 6 \times 3, \div 7$?
23. $5 \times 10 + 9 \times 9 - 59, \div 8$?

**II.** 1. $6 \times 4 + 9 + 9, \div 7 - 1, \times 8 - 4, \div 9, \times 6 =$ how many times 6?

2. $7 \times 5 - 3, \div 8, \times 9, \div 6 + 17 + 5, \div 7, \times 10 =$ how many times 5?

3. $8 \times 7 - 2, \div 6, \times 8 + 9, \div 9, \times 3 + 27, \div 6, \times 4 + 36 =$ how many times 8?

4. $9 \times 9 - 17, \div 8, + 8 \times 9, \div 10, \div 4, \times 2, \times 2, \times 7 + 3 + 9 + 4 =$ how many times 9?

5. $50 \div 10, \times 9 - 5, \div 8, \times 7 - 5, \div 6, \times 5 - 5, =$ how many times 5?

6. $6 \times 8 - 6, \div 6, \times 7 - 1, \div 6 + 2, \times 3 + 2, \div 4 =$ how many times 4?

7. $9 \times 7 + 5 + 4, \div 8 + 7 + 9 + 8 + 7 + 9, \div 7, \times 10 + 6 + 5, \div 9, \times 4 =$ how many times 6?

8. 8 × 5 — 4, ÷ 9 + 3, × 3 + 6, ÷ 9 + 4 × 5 + 7 = how many times 6 ?

9. 2 × 10 + 7 × 4, ÷ 8, × 6 — 3 × 7 + 21, ÷ 4, × 5 — 13 = how many times 8 ?

10. 5 × 9 + 9 × 3, ÷ 8 — 3, × 8 + 6 × 4, ÷ 9, × 7 — 9 — 5, ÷ 6, × 10 + 2 = how many times 9 ?

**I.** 1. What will 6 barrels of flour cost at 8 dollars per barrel ?

*Solution.* If one barrel cost 8 dollars, 6 barrels will cost 6 times 8 dollars, which are 48 dollars. Therefore, 48 dollars is the cost of 6 barrels of flour at 8 dollars per barrel.

2. What will 9 hats cost at 4 dollars apiece ?

3. What will 8 pounds of rice cost at 5 cents per pound ?

4. What will 7 drums of figs weigh if each drum weighs 6 pounds ?

5. How many quarts will 5 jugs hold if each jug holds 5 qt. ?

6. Sold 8 ploughs at 9 dollars apiece and 7 cultivators at 8 dollars apiece. How much did they come to ?

7. Bought 8 primers at 9 cents apiece, 7 small slates at 10 cents apiece, and 9 writing books at 7 cents apiece. What was the cost of the whole ?

8. Rode on Monday 8 hours at the rate of 7 miles per hour, on Tuesday 6 hours at the rate of 9 miles per hour, and on Wednesday 8 hours at the rate of 8 miles per hour. How far did I ride in all ?

9. 7 gallons = how many quarts ?

*Solution.* Since 1 gal. = 4 qts., 7 gal. must equal 7 times 4 quarts, or 28 quarts. Therefore, 7 gal. = 28 qt.

10. 9 gal. = how many quarts ? 8 weeks = how many days ?

11. 8 gal. = how many quarts ? 9 ℥ = how many ʒ ?

12. 9 bu. = how many pecks ? 5 pk. = how many quarts ?

13. 8 quarters = how many nails ? 6 weeks = how many days ?

14. 6 w. 4 d. = how many days ?

*Solution.* Since 1 week = 7 days, 6 weeks must equal 6 times 7 days, or 42 days, and 4 da. added are 46 days. Therefore, 6 w. 4 da. = 46 da.

15. 8 gal. 3 qt. = how many qt. ? 5 pk. 7 qt. = how many qt. ?

16. 9 yd. 2 ft. = how many ft. ? 4 ℥ 6 ʒ = how many ʒ ?

17. 4 m. 3 fur. = how many fur. ? 8 C. 5 cd. ft. = how many cord feet ?

**J.** 1. How many yards of cloth at 5 dollars per yard can be bought for 35 dollars?

*Solution.* If for 5 dollars 1 yard of cloth can be bought, for 35 dollars as many yards can be bought as there are times 5 in 35, which are 7 times. Therefore, 7 yards of cloth at 5 dollars per yard can be bought for 35 dollars.

2. How many bunches of crackers at 7 cents per bunch can be bought for 49 cents?

3. How many pounds of chalk at 4 cents per lb. can be bought for 24 cents?

4. If 3 cents will buy one roll of lozenges, how many rolls will 27 cents buy?

5. If 4 dollars will buy 1 pair of boots, how many pairs will 36 dollars buy?

6. If it takes a man 1 hour to walk 4 miles, how many hours will it take him to walk 28 miles?

7. If it takes a horse 1 hour to travel 9 miles, how many hours will it take to travel 72 miles?

8. If 1 hack will hold only 6 persons, how many hacks will it take to hold 54 persons?

9. 48 furlongs = how many miles?

*Solution.* Since 8 furlongs = 1 mile, 48 furlongs must equal as many miles as there are times 8 in 48, which are 6 times. Therefore, 48 furlongs = 6 miles.

10. 56 fur. = how many miles? 24 ft. = how many yd.?

11. 32 nails = how many qr.? 49 da. = how many weeks?

12. 40 pk. = how many bushels? 28 qt. = how many gallons?

13. 18 pt. = how many qt.? 24 ℈ = how many ʒ?

14. 28 farthings = how many pence? 35 dollars = how many half eagles?

15. 21 cents = how many three-cent pieces? 63 days = how many weeks?

16. 32 R. = how many acres? 27 sq. ft. = how many sq. yd.?

17. 28 gills = how many pt.? 28 da. = how many weeks?

18. 81 sq. ft. = how many sq. yd.? 80 pk. = how many bushels?

**K.** 1. How many jugs, each capable of containing 3 quarts, can be filled from 7 gal. 2 qt. of molasses?

*Solution.* Since 1 gal. = 4 qt., 7 gal. must equal 7 times 4 quarts, or 28 qt., and 2 quarts added are 30 quarts. Now, since each jug holds 3 quarts, as many jugs can be filled as there are times 3 qt. in 30 qt., which are 10 times. Therefore, 10 jugs can be filled from 7 gal. 2 qt. of molasses.

2. How many bags, each containing 5 pecks, can be filled from 8 bu. 3 pk. of meal?

3. How many house lots, each containing 1 A. 2 R. can be made from a piece of land containing 10 A. 2 R.?

4. How many pictures at 2d. 1 qr. each can be bought for 6d. 3 qr.?

5. How many times 9 days are there in 6 w. 3 da.?

6. James bought 9 yd. 1 ft. of twine, which he cut into pieces 2 yd. 1 ft. long. How many pieces did he have?

7. Sarah's mother offered to give her 8 large oranges, worth 5 cents apiece, if she would tell her how many 7 qt. baskets could be filled from 6 pk. 1 qt. of berries? Sarah answered correctly. What was her answer? She exchanged her oranges for drawing pencils worth 10 cents apiece. How many pencils did she get?

8. How many bushels in 8 bags, each containing 3 pk.?

9. How many pounds of rice at 6 cents a pound can be bought for 4 lb. of raisins at 9 cents per lb.?

10. How many casks of raisins at 8 dollars per cask can be bought for 6 cords of wood at 4 dollars per cord?

11. How many quarts of milk are there in 4 cans each containing 2 gallons?

12. Allen travelled on foot from Portsmouth to Portland, the distance being 51 miles. After travelling 6 hours at the rate of 4 miles per hour he stopped for the night. How far was he then from Portsmouth? from Portland? If he travels at the rate of 3 miles per hour the next day, how many hours must he travel on that day to finish his journey?

13. A boy who had 2 dimes, 1 half-dime, and 8 three-cent pieces, spent all his money for raisins at 7 cents per pound. How many pounds did he buy? He gave 1 lb. of raisins to his mother, and found that if he divided the rest equally among his schoolmates he would give them $\frac{1}{6}$ of a pound apiece. How many schoolmates had he?

14. A trader bought 48 yd. of cloth at $\frac{1}{8}$ of a dollar per yard. How many dollars did it cost him?

15. A fruit peddler paid 48 cents for cherries at 6 cts. per quart, which he put into papers containing $\frac{1}{4}$ of a quart each. How many papers did it take? He sold 8 of the papers at 4 cts. per paper, 9 for 3 cts. per paper, 10 for 2 cts. per paper, and the rest for 5 cts. per paper. How many cents did he receive for them?

16. If it takes 9 yards of silk to make 1 dress, how many dresses can be made from a piece of silk containing 36 yards? If the silk for each dress cost 8 dollars, what would the silk for them all cost?

17. A furniture dealer gave 6 bureaus worth 7 dollars apiece, and 3 dollars in money for chairs at 9 dollars per dozen. How many dozen chairs did he buy?

18. A laborer worked 6 weeks at 9 dollars per week, and, putting 6 dollars with the money thus earned, he bought coal at 6 dollars per ton. How many tons did he buy? After laying aside 2 tons of coal for his own use, he sold the remainder for $7\frac{1}{2}$ dollars per ton, receiving in payment for it 4 dollars in money and the rest in flour at 7 dollars per barrel. How many barrels of flour did he receive?

19. In Mr. Bacon's garden there are 9 rows of cucumbers with 7 hills in a row. In Mr. Ham's garden there are 9 hills more than in Mr. Bacon's, and they are arranged in rows of 9 hills each. How many rows of cucumbers in Mr. Ham's garden?

20. A fur dealer gave 8 caps at 5 dollars apiece and 2 dollars in money for muffs at 6 dollars apiece. How many muffs did he buy?

21. Augustus had 8 quarts of blackberries. He sold 1 quart of them for 11 cents, and the rest for 10 cents per quart. With the money he received for them he bought some cocoa nuts at 9 cents apiece. How many cocoa nuts did he buy?

22. He divided 2 of the cocoa nuts among his companions, and sold the rest for 10 cents apiece. He gave 7 cents of the money he thus received to a poor woman, and spent the rest for fire crackers at 7 cents per bunch. How many bunches did he buy?

23. Robert had a basket holding $7\frac{1}{3}$ quarts, and he gathered chestnuts enough to fill it 9 times full. How many peck baskets could he fill with what he gathered?

24. How many yards of vesting will it take to make 6 vests, if it take $\frac{3}{4}$ of a yard to make one vest?

25. How many aprons, each containing $\frac{6}{7}$ of a yard, can be made from $5\frac{1}{7}$ yards of calico?

26. How many yards of cloth at $1\frac{1}{5}$ dollars per yard can be bought for $9\frac{3}{5}$ dollars?

27. How many bushels of peaches at $1\frac{1}{8}$ dollars per bushel can be bought for $7\frac{7}{8}$ dollars?

28. How many hours must a man labor at $1\frac{4}{5}$ dimes per hour to earn $7\frac{1}{5}$ dimes?

29. Albert can run $\frac{6}{11}$ of a rod per second, and Jason can run $\frac{9}{11}$ of a rod in the same time. How many seconds will it take Albert to run as many rods as Jason can run in 4 seconds?

30. Mr. Jewett gave 24 lb. of candy to some boys, giving each boy 4 lb. To how many boys did he give the candy?

31. Mr. Reynolds gave $3\frac{3}{7}$ lb. of figs to some boys, giving $\frac{4}{7}$ of a pound to each boy. To how many boys did he give the figs?

32. Mr. Mussey divided 8 pine apples among some girls, giving $1\frac{1}{3}$ apples to each girl. Among how many girls did he divide the pine apples?

33. Spent $6\frac{2}{5}$ dollars for grain at $\frac{4}{5}$ dollar per bushel, and put the grain thus bought into bags each holding $1\frac{1}{3}$ bushels. How many bags did it fill?

34. A man paid $5\frac{2}{5}$ dimes for parched corn at $\frac{3}{5}$ of a dime per quart. After giving away 1 quart of the corn, he put the rest in bags each holding $\frac{4}{5}$ of a quart. How many bags did it take?

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## SECTION XI.

**A.**

1. 8 times what = 24? 6 times what = 36?
2. 6 times what = 42? 9 times what = 54?
3. 7 times what = 21? 5 times what = 45?
4. 8 times what = 56? 9 times what = 63?
5. 6 times what = 48? 4 times what = 24?
6. 4 times what = 32? 7 times what = 49?
7. 10 times what = 60? 10 times what = 70?
8. 8 times what = 40? 6 times what = 30?
9. 8 times what = 72? 9 times what = 81?
10. 9 times what = 27? 10 times what = 80?
11. 8 times what = 32? 8 times what = 64?
12. 4 times what = 36? 5 times what = 25?
13. 7 times what = 28? 7 times what = 35?

**B.** 1. What is $\frac{1}{4}$ of 24? *Answer.* $\frac{1}{4}$ of 24 is 6, because 4 times 6 = 24.

2. What is $\frac{1}{7}$ of 28? $\frac{1}{9}$ of 54? $\frac{1}{6}$ of 42? $\frac{1}{3}$ of 27?

3 What is $\frac{1}{6}$ of 18? $\frac{1}{7}$ of 63? $\frac{1}{8}$ of 48? $\frac{1}{8}$ of 56?

4. What is $\frac{4}{7}$ of 42?

*Solution.* $\frac{4}{7}$ of 42 = 4 times $\frac{1}{7}$ of 42. $\frac{1}{7}$ of 42 = 6, and $\frac{4}{7}$ of 42 must equal 4 times 6, which are 24. Therefore, $\frac{4}{7}$ of 42 = 24.

5. What is $\frac{4}{7}$ of 63? $\frac{6}{8}$ of 48? $\frac{7}{9}$ of 36? $\frac{5}{8}$ of 24?

6. What is $\frac{5}{9}$ of 27? $\frac{3}{7}$ of 28? $\frac{4}{5}$ of 25? $\frac{7}{8}$ of 56?

7. What is $\frac{3}{7}$ of 49? $\frac{5}{8}$ of 40? $\frac{2}{3}$ of 21? $\frac{7}{9}$ of 45?

8. What is $\frac{8}{9}$ of 81? $\frac{3}{7}$ of 63? $\frac{5}{8}$ of 64? $\frac{2}{3}$ of 27?

9. How many are $7\frac{1}{4}$ times 8?

*Solution.* $7\frac{1}{4}$ times 8 = 7 times 8, plus $\frac{1}{4}$ of 8; 7 times 8 = 56, and $\frac{1}{4}$ of 8 is 2, which added to 56 equals 58. Therefore, $7\frac{1}{4}$ times 8 = 58.

10. How many are $5\frac{1}{2}$ times 6? $8\frac{1}{3}$ times 9? $7\frac{2}{3}$ times 6?

11. How many are $6\frac{3}{7}$ times 7? $8\frac{4}{5}$ times 10? $6\frac{2}{3}$ times 9?

12. How many are $8\frac{3}{8}$ times 8? $4\frac{2}{3}$ times 6? $9\frac{3}{4}$ times 8?

13. What is $\frac{3}{4}$ of $\frac{8}{9}$?

*Solution.* $\frac{3}{4}$ of $\frac{8}{9}$ = 3 times $\frac{1}{4}$ of $\frac{8}{9}$; $\frac{1}{4}$ of $\frac{8}{9}$ = $\frac{2}{9}$, and 3 times $\frac{2}{9}$ = $\frac{6}{9}$. Therefore, $\frac{3}{4}$ of $\frac{8}{9}$ = $\frac{6}{9}$.

14. What is $\frac{2}{3}$ of $\frac{6}{7}$? $\frac{4}{5}$ of $\frac{10}{11}$? $\frac{5}{6}$ of $\frac{6}{7}$? $\frac{2}{3}$ of $\frac{9}{10}$?

15. What is $\frac{7}{9}$ of $\frac{18}{23}$? $\frac{2}{5}$ of $\frac{15}{17}$? $\frac{5}{9}$ of $\frac{27}{35}$? $\frac{2}{3}$ of $\frac{15}{29}$?

16. What is $\frac{1}{4}$ of $28\frac{8}{9}$?

*Suggestion.* $\frac{1}{4}$ of $28\frac{8}{9}$ = $\frac{1}{4}$ of 28 + $\frac{1}{4}$ of $\frac{8}{9}$.

17. What is $\frac{1}{7}$ of $21\frac{7}{9}$? $\frac{1}{3}$ of $15\frac{9}{10}$? $\frac{1}{9}$ of $63\frac{54}{79}$?

18. What is $\frac{3}{4}$ of $28\frac{8}{9}$? $\frac{6}{7}$ of $21\frac{7}{9}$? $\frac{2}{3}$ of $15\frac{9}{10}$?

19. What is $\frac{7}{9}$ of $63\frac{54}{79}$? $\frac{3}{7}$ of $49\frac{28}{37}$? $\frac{4}{5}$ of $25\frac{15}{16}$?

20. What is $\frac{5}{6}$ of $42\frac{54}{67}$? $\frac{5}{9}$ of $45\frac{72}{95}$? $\frac{3}{8}$ of $40\frac{64}{67}$?

21. What is $\frac{2}{5}$ of $45\frac{35}{39}$? $\frac{4}{7}$ of $28\frac{28}{39}$? $\frac{3}{5}$ of $15\frac{25}{31}$?

22. What is $\frac{2}{9}$ of $36\frac{81}{100}$? $\frac{5}{6}$ of $18\frac{12}{13}$? $\frac{3}{8}$ of $64\frac{56}{67}$?

23. What is $\frac{1}{7}$ of $5\frac{5}{6}$?

*Solution.* $5\frac{5}{6}$ = $\frac{35}{6}$, and $\frac{1}{7}$ of $\frac{35}{6}$ = $\frac{5}{6}$. Therefore, $\frac{1}{7}$ of $5\frac{5}{6}$ = $\frac{5}{6}$.

24. What is $\frac{2}{3}$ of $1\frac{4}{5}$? $\frac{3}{5}$ of $4\frac{2}{7}$? $\frac{2}{9}$ of $7\frac{5}{7}$?

25. What is $\frac{1}{4}$ of $2\frac{2}{3}$? $\frac{5}{7}$ of $6\frac{2}{9}$? $\frac{8}{9}$ of $7\frac{7}{8}$?

26. What is $\frac{7}{8}$ of $7\frac{1}{9}$? $\frac{1}{7}$ of $5\frac{3}{5}$? $\frac{5}{6}$ of $5\frac{8}{9}$?

27. What is $\frac{4}{5}$ of $4\frac{3}{8}$? $\frac{3}{4}$ of $3\frac{1}{9}$? $\frac{2}{3}$ of $2\frac{4}{7}$?

28. What is $\frac{1}{8}$ of $7\frac{1}{9}$? $\frac{5}{6}$ of $2\frac{4}{7}$? $\frac{3}{7}$ of $5\frac{2}{8}$?

29. $\frac{1}{6}$ of 42 + $\frac{1}{5}$ of 25? $\frac{2}{3}$ of 27 + $\frac{1}{7}$ of 49?

30. $\frac{4}{9}$ of 81 + $\frac{2}{7}$ of 56? $\frac{3}{7}$ of 28 + $\frac{2}{3}$ of 21?
31. $\frac{3}{4}$ of 36 + $\frac{3}{5}$ of 50? $\frac{2}{7}$ of 56 + $\frac{9}{10}$ of 60?
32. $\frac{4}{5}$ of 35 + $\frac{9}{10}$ of 50? $\frac{8}{9}$ of 45 + $\frac{6}{7}$ of 63?
33. $\frac{8}{9}$ of 72 + $\frac{4}{7}$ of 63? $\frac{8}{9}$ of 45 — $\frac{2}{3}$ of 27?
34. $\frac{4}{7}$ of 56 — $\frac{6}{7}$ of 28? $\frac{8}{9}$ of 81 + $\frac{9}{10}$ of 80?
35. $\frac{5}{8}$ of 64 + $\frac{6}{7}$ of 49? $\frac{3}{5}$ of 45 — $\frac{4}{7}$ of 42?
36. $\frac{3}{8}$ of 24 + $\frac{5}{6}$ of 48? $\frac{3}{8}$ of 72 + $\frac{4}{9}$ of 63?
37. $\frac{4}{7}$ of 42 + $\frac{7}{9}$ of 63? $\frac{7}{8}$ of 32 + $\frac{4}{7}$ of 28?
38. $\frac{4}{7}$ of 49 + $\frac{5}{6}$ of 54? $\frac{7}{8}$ of 48 — $\frac{3}{5}$ of 45?
39. $\frac{6}{7}$ of 70 — $\frac{2}{3}$ of 18? $\frac{5}{9}$ of 45 — $\frac{3}{7}$ of 21?

**C.** 1. $\frac{4}{5}$ of 30 + $\frac{2}{6}$ of 48, ÷ 10, × $\frac{3}{4}$ of 12, ÷ $\frac{1}{2}$ of 18, + $\frac{4}{7}$ of 21 = how many times 2?

2. $\frac{5}{9}$ of 45 + $\frac{7}{8}$ of 32, — $\frac{3}{5}$ of 30, ÷ $\frac{1}{7}$ of 49, × $\frac{3}{8}$ of 24, — 18 = how many times 9?

3. $\frac{3}{7}$ of 63 + $\frac{4}{9}$ of 81, ÷ 7, × $\frac{1}{8}$ of 72 — 9, ÷ 8, × $\frac{1}{9}$ of 36 + $\frac{3}{7}$ of 56 = how many times 6?

4. $\frac{6}{7}$ of 49 + $\frac{3}{4}$ of 36 + $\frac{5}{9}$ of 27 — 7 × 8, ÷ $\frac{1}{5}$ of 20 + 35 = how many times 7?

5. $\frac{4}{9}$ of 90, ÷ $\frac{1}{8}$ of 64, × $\frac{1}{10}$ of 70 + $\frac{7}{9}$ of 27, ÷ $\frac{1}{9}$ of 72, × 7, + 51 = how many times 10?

**D.** 1. What part of 4 is 1?

*Ans.* 1 is $\frac{1}{4}$ of 4, because 4 times 1 = 4.

2. What part of 6 is 1? of 9 is 1? of 7 is 1? of 3 is 1?

3. What part of 7 is 4?

*Solution.* Since 1 is $\frac{1}{7}$ of 7, 4 must be 4 times $\frac{1}{7}$ of 7, or $\frac{4}{7}$ of 7.

4. What part of 8 is 3? of 9 is 4? of 7 is 6? of 5 is 3?

5. What part of 9 is 7? of 7 is 9? of 5 is 4? of 10 is 7?

6. What part of 1 peck is 1 quart? is 3 qt.? is 7 qt.? is 9 qt.?

7. What part of 1 gal. is 1 qt.? is 3 qt.? is 2 qt. 1 pt., or 5 pt.? is 3 qt. 1 pt.?

8. What part of 1 gal. 3 qt., or 7 qt., is 3 qt.? is 1 gal.?

9. What part of 2 gal. 1 qt. is 2 qt.? is 1 gal.? is 1 gal. 3 qt.? is 2 gal.?

What part of 3 qt. is 1 qt? is 1 gal.? is 1 gal. 2 qt.?
.?

What part of 3 miles is 1 mile? of $\frac{3}{4}$ is $\frac{1}{4}$? of $\frac{3}{7}$ is $\frac{1}{7}$?
1 pt. is 1 pt.? of $1\frac{1}{2}$ is $\frac{1}{2}$?

What part of $\frac{8}{9}$ is $\frac{1}{9}$? of $1\frac{1}{7}$ is $\frac{3}{7}$? of 1 week 1 da. is 1
f $1\frac{3}{4}$ is $\frac{1}{4}$? of 1 gal. 3 qt. is 1 qt.?

What part of $\frac{7}{8}$ is $\frac{3}{8}$? of $\frac{8}{9}$ is $\frac{3}{9}$? of $\frac{5}{7}$ is $\frac{6}{7}$?

What part of $2\frac{1}{4}$ is $\frac{3}{4}$? of $1\frac{3}{7}$ is $\frac{4}{7}$? of $2\frac{1}{2}$ is $1\frac{1}{2}$?

What part of $4\frac{1}{5}$ is $2\frac{3}{5}$? of $2\frac{3}{5}$ is $4\frac{1}{5}$? of $7\frac{1}{3}$ is $2\frac{2}{3}$?

What part of $2\frac{2}{3}$ is $7\frac{1}{3}$? of $8\frac{5}{7}$ is $3\frac{2}{7}$? of $3\frac{2}{7}$ is $4\frac{1}{7}$?

What part of the cost of 7 books is the cost of one book?
oks? of 5 books?

What part of the value of 2 bu. 1 pk. is the value of 1
1 bu.? of 1 bu. 1 pk.? of 1 bu. 3 pk.? of 2 bu.?

A and B buy some goods for 8 dollars, of which A pays
rs and B pays 3. What part of the goods ought each
?

A and B trade in company; A puts in 4 dollars as often
its in 3. What part of the gain ought each to have?

A, B, and C traded together; A put in 2 dollars as often
it in 3, and C 4. What part of the whole did each put in?
*tion.* Since for every 2 dollars that A put in B put in
C 4, it is evident that for every 9 dollars that all put in,
in 2 dollars, B 3, and C 4. Therefore, A put in $\frac{2}{9}$, B
C $\frac{4}{9}$ of the whole.

A, B, and C hired a pasture together. For every cow
put in, B put in 2 and C 5. What part of the expense
pasture ought each one to pay?

1. If 7 sheep cost 49 dollars, what will 3 sheep cost?
*tion.* If 7 sheep cost 49 dollars, 1 sheep will cost $\frac{1}{7}$ of 49
, which is 7 dollars, and 3 sheep will cost 3 times 7 dol-
21 dollars. Therefore, 3 sheep will cost 21 dollars, if
p cost 49 dollars.

If 8 papers of candy cost 56 cents, how many cents will
rs cost?

If a girl receives 45 merit marks for 9 good lessons, how
will she receive for 4 good lessons?

If 3 milk-cans will hold 24 qts. of milk, how many
will 7 milk-cans hold?

If 9 boys can pick 54 quarts of blueberries in an after-
how many quarts can 7 boys pick in the same time?

6. How much will 3 quarts of molasses cost at 32 cents per gallon?

*Solution.* If 1 gallon, or 4 quarts, cost 32 cents, 1 qt. will cost $\frac{1}{4}$ of 32 cents, or 8 cents, and 3 quarts will cost 3 times 8 cents, or 24 cts. Therefore, 3 quarts of molasses at 32 cents per gallon will cost 24 cts.

7. How much will 7 quarts of meal cost at 24 cents per peck?

8. How much will 3 gills of lamp oil cost at 48 cents per quart?

9. If a yard of cloth is worth 24 cents, how much is a piece 2 ft. in length worth?

10. If 1 acre, 3 roods (or 7 roods) of woodland cost 63 dollars, how many dollars will 1 acre cost?

11. If 1 gallon and 1 pint of burning fluid cost 81 cents, how much will 1 quart cost? will 1 gallon cost?

12. If it cost 45 cents to dig a trench 1 yd. 2 ft. in length, how much will it cost to dig one 2 feet in length? How much to dig one 1 yd. 1 ft. in length?

13. If for 2 three-cent pieces and 2 cents, 64 marbles can be bought, how many can be bought for 1 three-cent piece and 2 cents? for 3 three-cent pieces?

14. Julia's basket holds 2 pt. 1 gi. If she can fill it with berries in 45 minutes, how many minutes will it take her to fill Susan's basket, which holds but 1 pint? How long to fill Josephine's, which holds but 3 gills?

15. What will $\frac{6}{7}$ of a pound of tea cost at 42 cents per pound?

*Solution.* If 1 pound of tea costs 42 cents, $\frac{1}{7}$ of a pound will cost $\frac{1}{7}$ of 42 cents, or 6 cents, and $\frac{6}{7}$ of a pound will cost 6 times 6 cents, or 36 cents. Therefore, $\frac{6}{7}$ of a pound of tea at 42 cents per pound will cost 36 cents.

16. If a man can perform a piece of work in 72 minutes, how many minutes will it take him to perform $\frac{5}{8}$ of it?

17. What will $\frac{5}{6}$ of a yard of linen cost at 54 cents per yard?

18. Deborah had 48 cents, and she gave $\frac{7}{8}$ of them for a doll. How many cents did the doll cost her?

19. Brass is composed of copper and zinc. If $\frac{1}{5}$ of it is composed of zinc and the rest of copper, how many pounds of copper will there be in a bar of brass weighing 25 lb.?

20. How much will $5\frac{3}{4}$ lb. of sugar cost at 8 cents per pound?

21. How much will $8\frac{2}{3}$ barrels of flour cost at 6 dollars per barrel?

22. If a man can walk 32 miles in 8 hours, how far can he walk in 1 hour? in 9$\frac{3}{4}$ hours?

23. If 8 tons meadow hay cost 72 dollars, how many dollars will 5$\frac{2}{3}$ tons cost?

24. If 7 yards of cloth are worth as much as 49 lb. of butter, how many pounds of butter ought 4$\frac{6}{7}$ yards of cloth to be worth?

25. Olive has 40 picture books, Ella has $\frac{4}{5}$ as many as Olive, and Ada has $\frac{7}{8}$ as many as Ella. How many has Ella? has Ada?

26. John is $\frac{1}{4}$ as old as his father, who is 36 years old, and William is $\frac{2}{3}$ as old as John. How old is John? is William?

27. It is 45 miles by rail-road from Worcester, Mass., to Nashua, N. H. W. Boylston is $\frac{1}{5}$ of this distance from Worcester, New Boston is $\frac{2}{5}$ of it, Harvard is $\frac{3}{5}$ of it, and Pepperell is $\frac{4}{5}$ of it. How far is each of the above places from Worcester? from Nashua?

28. It is 50 miles by the Fitchburg rail-road from Boston to Fitchburg. It is $\frac{1}{5}$ as far by the same road to Waltham, $\frac{2}{5}$ as far to Concord, $\frac{5}{10}$ as far to S. Acton, $\frac{7}{10}$ as far to Groton Junction, and $\frac{4}{5}$ as far to Shirley. How far is each of the above places from Boston? from Fitchburg?

29. I bought 3 pecks of corn at 40 cents per bushel, 3 pecks of oats at 36 cents per bushel, 5 quarts of walnuts at 48 cents per peck, 6 quarts of salt at 24 cents per peck, and 3 quarts of molasses at 28 cents per gallon. What was the amount of my purchase?

30. Edward had 3 cents and Robert had 5. They put their money together and bought 72 filberts. How many filberts ought each to have?

31. Harriet, Maria, and Caroline, send some berries to market, for which they receive 63 cents. Now if Harriet sent 3 qt., Maria 2, and Caroline 4, how many cents ought each in justice to receive?

32. Joseph Fisher and Reuben Guild buy a horse together, towards which Fisher pays 5 dollars as often as Guild pays 2. They sell him so as to gain 56 dollars. What will be each man's share of the gain?

33. Hiram picked 1 qt. 1 pt. of strawberries, Arthur picked 2 qts., and Rowland picked 1 qt. They put them all together and sold them for 81 cents. How many cents should each receive?

## SECTION XII.

**A.** 1. Any number which is an exact number of times a given number, is a multiple of that given number. Thus 21 is a multiple of 3, because it equals 7 times 3.

2. The multiples of a number are those numbers which would be produced by successive additions of the given number to itself. Thus, 2, 4, 6, 8, 10, 12, &c., are multiples of 2; 7, 14, 21, 28, &c. are multiples of 7.

3. What are the first 10 multiples of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10?

4. *Of what numbers is 24 a multiple? is 20? 21? 36? 30? 28? 25? 32? 35? 40? 42? 45? 48? 50? 54? 56? 60? 64? 70? 72? 80? 81? 90?

5. What is the largest multiple of 4 less than the number 23? *Ans.* 20, which is 5 times 4.

6. What is the largest multiple of 5 less than the number 26? of 3? of 7? of 4? of 8? of 9? of 6? of 10?

7. What is the largest multiple of 6 less than the number 35? of 8? of 4? of 10? of 9?

8. What is the largest multiple of 8 less than the number 43? of 7? of 4? of 9? of 8? of 5? of 6?

9. What is the largest multiple of 7 less than the number 73? of 10? of 9? of 8?

10. What is the largest multiple of 6 less than the number 51? of 9? of 7? of 10? of 5? of 8?

11. What is the largest multiple of 4 less than the number 29? of 7? of 6? of 9? of 8? of 3? of 5? of 10?

12. What is the largest multiple of 6 less than the number 62? of 8? of 10? of 9? of 7?

13. What is the largest multiple of 7 less than the number 39? of 4? of 6? of 7? of 5? of 9? of 10? of 8?

**B.** 1. 25 = how many times 7?

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* In these questions it should be borne in mind that the pupil has not yet been led to multiply or divide by any number larger than 10. He should not therefore be required to mention any factor greater than 10, or any factor which is contained in the number considered more than 10 times.

*Solution* 1*st*. 25 = 3 times 7, and 4 over, because 3 times 7 = 21, and 4 added = 25.

*Solution* 2*nd*. 25 = $3\frac{4}{7}$ times 7, because 3 times 7 = 21, and $\frac{4}{7}$ of 7, which are 4, added to 21 = 25.

NOTE. It is essential that the pupil become perfectly familiar with both the above forms of solution. The best method will doubtless be to have him use one form till he masters it, and then use the other. In the first form the 4 should be regarded as an *undivided number* or *remainder*, but in the second, the $\frac{4}{7}$ should be regarded as a *part of the quotient*, and *not* as a remainder.

2. 26 = how many times 5? 9? 3? 8? 10? 6? 4?
3. 33 = how many times 6? 9? 4? 7? 5? 8? 10?
4. 37 = how many times 4? 8? 6? 10? 5? 7? 9?
5. 41 = how many times 9? 10? 7? 5? 4? 8? 6?
6. 83 = how many times 10? 8? 9?
7. 72 = how many times 8? 9? 10? 7?
8. 63 = how many times 6? 7? 8? 9? 10?
9. 57 = how many times 7? 10? 8? 6? 9?
10. 29 = how many times 10? 9? 3? 5? 7? 4? 8? 6?
11. 48 = how many times 8? 7? 10? 5? 6? 9?
12. 35 = how many times 4? 7? 10? 9? 6? 5? 8?
13. 46 = how many times 9? 5? 8? 7? 6? 10?
14. 53 = how many times 9? 5? 7? 8? 10? 6?
15. 65 = how many times 6? 8? 7? 10? 9?
16. 21 = how many times 7? 3? 9? 2? 8? 5? 4? 6? 10?
17. 39 = how many times 10? 9? 8? 7? 6? 5? 4?
18. 51 = how many times 8? 9? 10? 6? 7? 5?
19. 42 = how many times 6? 9? 4? 7? 5? 8? 10?
20. 28 = how many times 9? 10? 4? 6? 7? 3? 5? 8?
21. 62 = how many times 10? 8? 6? 7? 9?
22. 76 = how many times 7? 9? 10? 8?
23. 59 = how many times 6? 10? 9? 7? 8?
24. 45 = how many times 10? 5? 6? 9? 8? 7?

**C.** 1. Reduce $\frac{29}{6}$ to ones.

*Solution.* Since 6 sixths = 1, $\frac{29}{6}$ must equal as many ones as there are times 6 sixths in 29 sixths, which are $4\frac{5}{6}$ times. Therefore, $\frac{29}{6}$ = $4\frac{5}{6}$.

2. Reduce to ones, $\frac{43}{7}$; $\frac{64}{9}$; $\frac{83}{10}$; $\frac{27}{5}$; $\frac{45}{6}$; $\frac{87}{10}$; $\frac{45}{8}$; $\frac{28}{3}$.
3. Reduce to ones, $\frac{56}{6}$; $\frac{39}{4}$; $\frac{96}{9}$; $\frac{46}{9}$; $\frac{33}{4}$; $\frac{51}{7}$; $\frac{23}{5}$; $\frac{81}{10}$.
4. Reduce to ones, $\frac{59}{7}$; $\frac{79}{8}$; $\frac{37}{8}$; $\frac{20}{3}$; $\frac{41}{7}$; $\frac{47}{7}$; $\frac{19}{2}$; $\frac{14}{3}$.
5. Reduce to ones, $\frac{68}{7}$; $\frac{27}{8}$; $\frac{39}{5}$.

**D.** 1. How many are 7 times $\frac{8}{9}$?

*Solution.* 7 times $\frac{8}{9} = \frac{56}{9} = 6\frac{2}{9}$.

2. How many are 5 times $\frac{4}{7}$? 9 times $\frac{3}{4}$? 5 times $\frac{7}{8}$?
3. How many are 6 times $\frac{3}{5}$? 8 times $\frac{3}{10}$? 8 times $\frac{3}{7}$?
4. How many are 8 times $\frac{3}{4}$? 9 times $\frac{7}{10}$? 9 times $\frac{7}{8}$?
5. How many are 7 times $\frac{8}{9}$? 8 times $\frac{9}{11}$? 4 times $\frac{7}{8}$?
6. How many are 6 times $7\frac{4}{5}$?

*Solution.* 6 times $7\frac{4}{5} =$ 6 times $7 +$ 6 times $\frac{4}{5}$; 6 times $7 = 42$, 6 times $\frac{4}{5} = \frac{24}{5}$ or $4\frac{4}{5}$, which added to 42 are $46\frac{4}{5}$. Therefore, 6 times $7\frac{4}{5} = 46\frac{4}{5}$.

7. How many are 8 times $4\frac{3}{7}$? 9 times $7\frac{5}{8}$? 6 times $8\frac{7}{10}$?
8. How many are 10 times $3\frac{4}{5}$? 6 times $5\frac{8}{9}$? 3 times $7\frac{2}{5}$?
9. How many are 8 times $9\frac{4}{5}$? 9 times $7\frac{6}{7}$? 4 times $5\frac{3}{7}$?
10. How many are 8 times $9\frac{2}{3}$? 3 times $8\frac{7}{8}$? 5 times $8\frac{3}{4}$?
11. How many are 7 times $6\frac{4}{9}$? 9 times $7\frac{7}{8}$? 4 times $9\frac{4}{5}$?
12. How many are 8 times $7\frac{7}{8}$? 10 times $7\frac{6}{7}$? 6 times $7\frac{5}{8}$?

**E.** 1. $4\frac{1}{7} =$ how many times $\frac{3}{7}$?

*Solution.* $4\frac{1}{7} = \frac{29}{7}$, and $\frac{3}{7}$ is contained $9\frac{2}{3}$ times in $\frac{29}{7}$. Therefore, $4\frac{1}{7} = 9\frac{2}{3}$ times $\frac{3}{7}$.

2. $3\frac{5}{8} =$ how many times $\frac{7}{8}$? $9\frac{2}{7} =$ how many times $\frac{6}{7}$?
3. $6\frac{3}{8} =$ how many times $1\frac{1}{8}$ or $\frac{9}{8}$? $4\frac{7}{9} =$ how many times $\frac{5}{9}$?
4. $9\frac{3}{5} =$ how many times $1\frac{3}{5}$? $7\frac{1}{6} =$ how many times $\frac{5}{6}$?
5. $8\frac{4}{6} =$ how many times $1\frac{1}{6}$? $8\frac{3}{7} =$ how many times $1\frac{2}{7}$?
6. $9\frac{2}{3} =$ how many times $2\frac{2}{3}$? $8\frac{3}{4} =$ how many times $2\frac{1}{4}$?
7. $7\frac{6}{7} =$ how many times $1\frac{1}{7}$? $5\frac{5}{6} =$ how many times $1\frac{3}{6}$?
8. $8\frac{1}{4} =$ how many times $2\frac{1}{4}$? $6 =$ how many times $\frac{7}{9}$?

**F.** 1. Moses has 35 cents, with which he wishes to buy oranges at 6 cents apiece. How many oranges can he buy?

*Solution.* If for 6 cents he can buy one orange, for 35 cents he can buy as many oranges as there are times 6 cents in 35 cents, which are $5\frac{5}{6}$ times, or 5 times with a remainder of 5 cents. Therefore he can buy $5\frac{5}{6}$ oranges, or he can buy 5 oranges and have 5 cents left.

2. How many barrels of flour at 6 dollars per barrel can be bought for 58 dollars?

3. How many shawls at 7 dollars apiece can be bought for 39 dollars?

4. How many toys at 9 cents each can be bought for 52 cents?

5. How many books at 3 dollars apiece can be bought for 29 dollars?

6. How many terms' tuition at 7 dollars per term will 27 dollars pay for?

7. How many shade-trees at $\frac{4}{7}$ of a dollar apiece can be paid for with $5\frac{3}{7}$ dollars?

8. How many skeins of silk at $\frac{4}{5}$ of a dime per skein can be paid for with $5\frac{1}{5}$ dimes?

9. How many baskets, each containing $\frac{6}{7}$ of a bushel, can be filled from $8\frac{3}{7}$ bushels of peaches?

10. How many boxes, each containing $\frac{7}{8}$ of a quart, can be filled from $7\frac{5}{8}$ quarts of blackberries?

11. How many pounds of sugar at 8 cents per lb. can be bought for $7\frac{2}{3}$ dozen of eggs at 9 cents per dozen?

12. How many pounds of rice at 6 cents per lb. can be obtained for $8\frac{3}{5}$ quarts of milk at 5 cents per quart?

13. Bought $7\frac{1}{4}$ yards of cloth at 4 dollars per yard, which I paid for with cider at 3 dollars per barrel. How many barrels did it take?

14. Gave $8\frac{3}{4}$ quarts of cherries at 8 cents per quart for sheeting at 9 cents per yard. How many yards of sheeting ought I to receive?

15. How many three-quart jugs can be filled from 8 gallons of molasses?

16. How many six-quart baskets can be filled from $7\frac{1}{4}$ pecks of apples?

17. A tailor paid 45 dollars for silk velvet at 5 dollars per yard. He made the velvet into vests, putting $\frac{5}{6}$ of a yard into each vest. How many vests did he make, and how many yards did he have remaining?

18. One "Fourth of July," Thomas had 29 cents to spend. He bought as many bunches of crackers at 10 cents per bunch as he could pay for, and then spent the rest of his money for cherries at the rate of 7 for a cent. How many bunches of crackers did he buy? How many cherries?

19. Richard worked 8 days for $3\frac{3}{4}$ dimes per day, 7 days for $5\frac{1}{2}$ dimes per day, and 9 days for $2\frac{1}{2}$ dimes per day. He paid 26 dimes for board, and gave the rest to a trader for cloth at 9 dimes per yard. How many yards did he buy?

20. A farmer exchanged 5 barrels of apples at $1\frac{6}{7}$ dollars per barrel, for oil at $1\frac{1}{7}$ dollars per gallon. How many gallons of oil did he receive? He afterwards gave 4 gallons of the oil for tea at $\frac{3}{7}$ of a dollar per lb. How many pounds of tea did he buy? He used $2\frac{2}{3}$ lb. of the tea, and exchanged the rest for syrup at $\frac{5}{7}$ of a dollar per gallon. How many gallons of syrup did he get?

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## SECTION XIII.

**A.** 1. What is $\frac{1}{8}$ of 29?

*Solution.* $\frac{1}{8}$ of 29 $=$ $\frac{1}{8}$ of 24 $+$ $\frac{1}{8}$ of 5 $= 3\frac{5}{8}$.

2. What is $\frac{1}{7}$ of 36? $\frac{1}{5}$ of 28? $\frac{3}{5}$ of 28? $\frac{2}{9}$ of 47?

3. What is $\frac{3}{4}$ of 35? $\frac{4}{7}$ of 58? $\frac{3}{8}$ of 79? $\frac{4}{5}$ of 48?

4. What is $\frac{5}{6}$ of 53? $\frac{6}{7}$ of 76? $\frac{3}{8}$ of 62? $\frac{7}{9}$ of 88?

5. What is $\frac{5}{8}$ of 31? $\frac{5}{9}$ of 47? $\frac{6}{7}$ of 48? $\frac{2}{3}$ of 26?

6. What is $\frac{2}{5}$ of 28? $\frac{3}{7}$ of 60? $\frac{4}{9}$ of 33? $\frac{5}{8}$ of 63?

7. What is $\frac{5}{6}$ of 56? $\frac{2}{3}$ of 29? $\frac{3}{10}$ of 99? $\frac{5}{7}$ of 55?

**B.** 1. $\frac{7}{9}$ of 36 $=$ how many times 5?

2. $\frac{3}{5}$ of 25 $=$ how many times 2?

3. $\frac{7}{10}$ of 70 $=$ how many times 5?

4. $\frac{8}{7}$ of 49 $=$ how many times 8?

5. $\frac{4}{5}$ of 45 $=$ how many times 8?

6. $\frac{3}{8}$ of 63 $=$ how many times 6?

7. $\frac{4}{7}$ of 56 $=$ how many times 9?

8. $\frac{9}{8}$ of 64 $=$ how many times 7?

9. $\frac{3}{5}$ of 35 $+$ $\frac{4}{9}$ of 81 $=$ how many times 6?

10. $\frac{5}{9}$ of 45 $+$ $\frac{3}{5}$ of 45 $=$ how many times 8?

11. $\frac{4}{7}$ of 25 $+$ $\frac{3}{7}$ of 60 $=$ how many times 7?

12. $\frac{2}{3}$ of 19 $+$ $\frac{2}{3}$ of 29 $=$ how many times 4?

**C.** 1. $\frac{8}{9}$ of 45 = how many times $\frac{1}{6}$ of 42?

*Solution.* $\frac{1}{9}$ of 45 = 5, and $\frac{8}{9}$ of 45 must equal 8 times 5 or 40; $\frac{1}{6}$ of 42 = 7, and 7 is contained in 40, $5\frac{5}{7}$ times. Therefore, $\frac{8}{9}$ of 45 = $5\frac{5}{7}$ times $\frac{1}{6}$ of 42.

2. $\frac{7}{10}$ of 80 = how many times $\frac{1}{8}$ of 64?
3. $\frac{5}{6}$ of 36 = how many times $\frac{1}{8}$ of 24?
4. $\frac{6}{8}$ of 72 = how many times $\frac{1}{4}$ of 32?
5. $\frac{9}{5}$ of 40 = how many times $\frac{5}{6}$ of 12?
6. $\frac{5}{2}$ of 16 = how many times $\frac{2}{3}$ of 9?
7. $\frac{2}{7}$ of 25 = how many times $\frac{1}{7}$ of 9?
8. $\frac{3}{8}$ of 19 = how many times $\frac{1}{6}$ of 7?
9. $\frac{4}{5}$ of 14 = how many times $\frac{1}{5}$ of 8?
10. $\frac{2}{3}$ of 13 = how many times $\frac{1}{2}$ of 10?

**D.** 1. What is $\frac{1}{7}$ of $61\frac{4}{9}$?

*Solution.* $\frac{1}{7}$ of $61\frac{4}{9} = \frac{1}{7}$ of $56 + \frac{1}{7}$ of $5\frac{4}{9}$ or $\frac{49}{9} = 8 + \frac{7}{9} = 8\frac{7}{9}$.

2. What is $\frac{1}{5}$ of $38\frac{1}{8}$? $\frac{1}{8}$ of $37\frac{5}{7}$? $\frac{1}{9}$ of $78\frac{6}{8}$?
3. What is $\frac{1}{6}$ of $47\frac{1}{7}$? $\frac{1}{3}$ of $26\frac{5}{8}$? $\frac{3}{4}$ of $35\frac{3}{7}$?
4. What is $\frac{5}{8}$ of $46\frac{2}{9}$? $\frac{2}{5}$ of $47\frac{2}{9}$? $\frac{4}{7}$ of $60\frac{4}{6}$?
5. What is $\frac{5}{6}$ of $58\frac{2}{7}$? $\frac{2}{3}$ of $23\frac{2}{5}$? $\frac{8}{9}$ of $79\frac{7}{8}$?
6. What is $\frac{3}{8}$ of $39\frac{1}{9}$? $\frac{4}{9}$ of $43\frac{5}{7}$? $\frac{7}{10}$ of $48\frac{6}{8}$?
7. What is $\frac{6}{7}$ of $13\frac{2}{9}$? $\frac{4}{5}$ of $8\frac{6}{8}$? $\frac{2}{3}$ of $29\frac{5}{8}$?
8. What is $\frac{3}{4}$ of $19\frac{5}{9}$? $\frac{5}{6}$ of $65\frac{1}{7}$? $\frac{7}{8}$ of $21\frac{1}{3}$?

**E.** 1. If 7 inkstands cost 45 cents what will 3 cost?

2. If 9 melons cost 77 cents what will 5 cost?

3. If 8 weeks' board cost 27 dollars what will 7 cost?

4. If 4 men eat 23 pounds of meat in a month, how many pounds will 7 men eat in the same time?

5. If 8 horses eat 37 cwt. of hay in 1 month, how much will 5 horses eat in the same time?

6. If a peck of cranberries is worth 53 cents, how many cents are 3 quarts worth?

7. If a gallon of burning fluid is worth 79 cents, what are 1 qt. and 1 pint worth?

8. $\frac{5}{8}$ of a furlong = how many rods?

*Suggestion.* Since 1 furlong = 40 rods, $\frac{5}{8}$ of a furlong must equal $\frac{5}{8}$ of 40 rods.

9. $\frac{4}{5}$ of a qr. = how many lb.?

10. $\frac{6}{7}$ of a cu. yd. = how many cu. ft.?

11. $\frac{5}{9}$ of an hour = how many minutes?

12. $\frac{4}{9}$ of a day = how many hours?

13. $\frac{7}{10}$ of a R. = how many sq. rd.?

14. $\frac{5}{6}$ of a £ = how many shillings?

15. $\frac{3}{7}$ of a ton = how many cwt.?

16. $\frac{3}{5}$ of a peck = how many qts.?

17. $\frac{2}{3}$ of a bushel = how many pecks, quarts, &c.?

*Solution.* Since 1 bu. = 4 pks., $\frac{2}{3}$ of a bushel must equal $\frac{2}{3}$ of 4 pecks, or $2\frac{2}{3}$ pecks. But since 1 pk. = 8 qts., $\frac{2}{3}$ of a peck must equal $\frac{2}{3}$ of 8 quarts, or $5\frac{1}{3}$ qts. Since 1 qt. = 2 pt., $\frac{1}{3}$ of a quart must equal $\frac{1}{3}$ of 2 pints, or $\frac{2}{3}$ of a pint. Since 1 pt. = 4 gills, $\frac{2}{3}$ of a pint must equal $\frac{2}{3}$ of 4 gills, or $2\frac{2}{3}$ gills. $\frac{2}{3}$ of a bushel is therefore equal to 2 pk. 5 qt. 0 pt. $2\frac{2}{3}$ gills.

18. $\frac{5}{7}$ of a £ = how many s. d. and qrs.?

19. $\frac{4}{5}$ of a lb. Troy = how many oz. and dwt.?

20. $\frac{6}{7}$ of a ton = how many cwt. qr. and lb.?

21. $\frac{5}{8}$ of a sq. yd. = how many sq. ft. and sq. in.?

22. $\frac{5}{7}$ of a lb. = how many oz. dwt. and gr.?

23. $\frac{4}{9}$ of a week = how many da. h., &c.?

24. $\frac{3}{5}$ of a ℔. = how many ℥, ʒ, &c.?

25. $\frac{7}{9}$ of a £ = how many s. d. and qr.?

26. $\frac{3}{5}$ of a s. = how many d. and qr.?

27. $\frac{4}{5}$ of a cwt. = how many qr. lb., &c.?

28. $\frac{2}{9}$ of a bu. = how many pk. qt., &c.?

29. $\frac{7}{10}$ of a mile = how many fur. rd., &c.?

30. If 5 lb. of brown sugar cost as much as 9 lb. of molasses sugar at $4\frac{1}{3}$ cents per lb., how much will 9 lb. of brown sugar cost?

31. If it takes $7\frac{3}{4}$ yards of calico to make 1 dress, and calico enough for 8 dresses can be bought for 7 dollars, how many yards can be bought for 5 dollars?

*Suggestion.* First find how many yards can be bought for 7 dollars.

32. If it takes $5\frac{5}{6}$ yds. of sheeting to make one sheet, and sheeting enough to make 6 sheets can be bought for 4 dollars, how many yards can be bought for 7 dollars?

33. Mr. Presby and Mr. Wheelock agreed to chop a lot of wood for 61 dollars, and share the money according to the work

that each did. On trial they found that Mr. Presby could cut only 3 cords while Mr. Wheelock cut 5. What part of the whole would each man cut? How many dollars ought each man to receive?

34. Frederic and Benjamin gathered some nuts, of which Frederic gathered 4 quarts and Benjamin 2 quarts. They sold them for 39 cents. How many cents ought each to receive?

35. James, wishing to buy some drawing-pencils, asked his father for the money. His father replied, "5 good drawing-pencils will cost as much as 7 writing-books at 7 cents apiece. Now, if you can tell me what six drawing-pencils will cost I will buy them for you." What should have been the answer?

36. Mr. Ames and Mr. Clapp bought the apples on 2 large trees, for 9 dollars, of which Mr. Ames paid 5 dollars and Mr. Clapp 4 dollars. There proved to be 87 pecks of apples. How many pecks ought each to have?

37. Waldo and George undertook to dig a large field of potatoes for 24 dollars. It turned out that Waldo could dig 3 bushels as often as George could dig 2. What part of the whole did each dig? They divided the money in proportion to the work each did. How many dollars did each receive?

*Partial explanation.* Since Waldo dug 3 bushels as often as George dug 2, he would dig 3 bushels out of every 5 that were dug, and would, therefore, dig $\frac{3}{5}$ of the whole, while George would dig $\frac{2}{5}$ of the whole.

38. Mr. Bacon, Mr. Calrow and Mr. Hawley entered into a speculation together; Mr. Bacon put in 4 dollars as often as Mr. Calrow put in 3, and as often as Mr. Hawley put in 2. What part of the whole stock did each man put in? They gained 75 dollars. How many dollars ought Mr. Bacon to receive on his share? ought Mr. Calrow? ought Mr. Hawley?

39. A and B traded together; B put in twice as much money as A, and both put in 28 dollars. How much did each man put in?

*Partial solution.* Since B put in twice as much as A, A and B together must have put in 3 times as much as A alone. Therefore, 28 dollars is 3 times what A put in. If 28 dollars is 3 times what A put in, A must have put in $\frac{1}{3}$ of 28 dollars, which is, &c.

40. Mr. Hooker and Mr. Phillips hired a pasture for 49 dollars; Mr. Hooker pastured 4 times as many animals as Mr. Phillips. How many dollars ought each to pay?

41. Mr. Bates and Mr. Conant received 33 dollars for doing a job of work, but Mr. Bates had done 3 times as much as Mr. Conant. How much ought each to receive?

42. William and Henry were talking about their ages. William said, "I am 8 years old." Then said Henry, "I am $7\frac{2}{3}$ years more than $\frac{2}{3}$ as old as you are, and my father is $\frac{10}{3}$ as old as I am. Now, if you will tell me how old I am, I will give you $\frac{6}{7}$ of 26 oranges, and if you will tell me how old my father is, I will give you $\frac{3}{8}$ of 48 apples." What would you have called Henry's age, had you been in William's place? What would you have called his father's age? William gave both answers correctly, and Henry gave him the oranges and apples as he had promised him. How many oranges did William receive? How many apples?

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## SECTION XIV.

**A.** 1. $3 = \frac{1}{8}$ of what number?

*Ans.* $3 = \frac{1}{8}$ of 8 times 3 or 24.

2. $9 = \frac{1}{5}$ of what number? $8 = \frac{1}{7}$ of what number?
3. $7 = \frac{1}{8}$ of what number? $9 = \frac{1}{4}$ of what number?
4. $6 = \frac{1}{9}$ of what number? $7 = \frac{1}{7}$ of what number?
5. $5 = \frac{1}{7}$ of what number? $9 = \frac{1}{3}$ of what number?
6. $\frac{8}{9} = \frac{1}{7}$ of what number?

*Solution.* $\frac{8}{9} = \frac{1}{7}$ of 7 times $\frac{8}{9}$, which are $\frac{56}{9}$, equal to $6\frac{2}{9}$ Therefore, $\frac{8}{9} = \frac{1}{7}$ of $6\frac{2}{9}$.

7. $\frac{5}{6} = \frac{1}{8}$ of what number? $\frac{9}{10} = \frac{1}{7}$ of what number?
8. $\frac{8}{9} = \frac{1}{3}$ of what number? $\frac{6}{7} = \frac{1}{4}$ of what number?
9. $\frac{7}{10} = \frac{1}{9}$ of what number? $\frac{7}{8} = \frac{1}{9}$ of what number?
10. $\frac{4}{5} = \frac{1}{8}$ of what number? $\frac{3}{7} = \frac{1}{6}$ of what number?
11. $9\frac{4}{7} = \frac{1}{8}$ of what number? $3\frac{7}{8} = \frac{1}{6}$ of what number?
12. $4\frac{5}{7} = \frac{1}{6}$ of what number? $8\frac{5}{8} = \frac{1}{6}$ of what number?
13. $7\frac{2}{5} = \frac{1}{4}$ of what number? $9\frac{2}{3} = \frac{1}{9}$ of what number?

**B.** 1. $24 = \frac{3}{7}$ of what number?

*Solution.* If 24 is $\frac{3}{7}$ of some number, $\frac{1}{7}$ of that number must be $\frac{1}{3}$ of 24, which is 8; and $\frac{7}{7}$, or the number, must be 7 times 8, which are 56. Therefore, $24 = \frac{3}{7}$ of 56.

2. $36 = \frac{4}{7}$ of what number? $25 = \frac{5}{7}$ of what number?
3. $33 = \frac{4}{9}$ of what number? $27 = \frac{7}{8}$ of what number?
4. $43 = \frac{5}{8}$ of what number? $81 = \frac{8}{9}$ of what number?
5. $45 = \frac{6}{7}$ of what number? $63 = \frac{8}{9}$ of what number?

6. $25 = \frac{7}{10}$ of what number? $21 = \frac{4}{9}$ of what number?
7. $\frac{63}{95} = \frac{7}{8}$ of what number? $\frac{27}{47} = \frac{3}{4}$ of what number?
8. $\frac{36}{77} = \frac{4}{5}$ of what number? $\frac{42}{83} = \frac{6}{7}$ of what number?
9. $24\frac{48}{59} = \frac{6}{7}$ of what number?
10. $54\frac{18}{37} = \frac{9}{10}$ of what number?
11. *$81\frac{27}{61} = 2\frac{1}{4}$ times what number?
12. $64\frac{72}{77} = 1\frac{3}{5}$ times what number?
13. $2\frac{7}{9} = 2\frac{1}{2}$ times what number?
14. $5\frac{5}{8} = 1\frac{4}{5}$ times what number?
15. $6\frac{2}{9} = 2\frac{2}{3}$ times what number?
16. $2\frac{7}{10} = \frac{3}{4}$ times what number?
17. $1\frac{15}{49} = 1\frac{3}{5}$ times what number?
18. $1\frac{17}{19} = 4\frac{1}{2}$ times what number?

**C.** 1. $46\frac{3}{7} = \frac{5}{8}$ of what number?

*Solution.* If $46\frac{3}{7} = \frac{5}{8}$ of some number, $\frac{1}{8}$ of that number must be $\frac{1}{5}$ of $46\frac{3}{7}$. $\frac{1}{5}$ of $45 = 9$, and $\frac{1}{5}$ of $1\frac{3}{7}$ or $\frac{10}{7} = \frac{2}{7}$, which added to 9 equals $9\frac{2}{7}$. If $9\frac{2}{7}$ is $\frac{1}{8}$ of a number, $\frac{8}{8}$ or the number must be 8 times $9\frac{2}{7}$, which are $72\frac{16}{7} = 74\frac{2}{7}$. Therefore, $46\frac{3}{7} = \frac{5}{8}$ of $74\frac{2}{7}$.

2. $18\frac{2}{5} = \frac{4}{7}$ of what number?
3. $58\frac{4}{5} = \frac{7}{10}$ of what number?
4. $9\frac{1}{7} = \frac{2}{3}$ of what number?
5. $37\frac{7}{9} = \frac{5}{6}$ of what number?
6. $32\frac{12}{43} = \frac{4}{9}$ of what number?
7. $37\frac{6}{7} = \frac{5}{9}$ of what number?
8. $67\frac{5}{9} = \frac{3}{5}$ of what number?
9. $4\frac{4}{5} = \frac{8}{9}$ of what number?
10. $86\frac{5}{8} = \frac{9}{10}$ of what number?
11. $49\frac{63}{136} = \frac{7}{8}$ of what number?
12. $9\frac{1}{3} = 1\frac{1}{3}$ times or $\frac{4}{3}$ of what number?
13. $71\frac{17}{25} = 1\frac{3}{4}$ times what number?
14. $57\frac{2}{61} = 1\frac{2}{5}$ times what number?
15. $46\frac{2}{9} = 1\frac{1}{7}$ times what number?
16. $38\frac{7}{10} = 1\frac{4}{5}$ times what number?
17. $25\frac{18}{29} = 2\frac{2}{3}$ times what number?

* This question is equivalent to, $81\frac{27}{61} = \frac{9}{4}$ of what number?

**D.** 1. $\frac{6}{7}$ of 42 $=$ $\frac{4}{9}$ of what number?

*Solution.* $\frac{1}{7}$ of 42 $=$ 6, and $\frac{6}{7}$ of 42 must equal 6 times 6, or 36. If 36 is $\frac{4}{9}$ of some number, $\frac{1}{9}$ of that number must equal $\frac{1}{4}$ of 36, which is 9, and $\frac{9}{9}$ or the number must equal 9 times 9, or 81. Therefore, $\frac{6}{7}$ of 42 $=$ $\frac{4}{9}$ of 81.

2. $\frac{8}{9}$ of 27 $=$ $\frac{4}{7}$ of what number?
3. $\frac{7}{9}$ of 72 $=$ $\frac{5}{6}$ of what number?
4. $\frac{4}{7}$ of 70 $=$ $\frac{5}{8}$ of what number?
5. $\frac{3}{7}$ of 42 $=$ $\frac{4}{9}$ of what number?
6. $\frac{5}{8}$ of 56 $=$ $\frac{9}{10}$ of what number?
7. $\frac{4}{9}$ of 54 $=$ $\frac{3}{8}$ of what number?
8. $\frac{2}{3}$ of 27 $=$ $\frac{3}{8}$ of what number?
9. $\frac{5}{4}$ of 36 $=$ $\frac{3}{4}$ of what number?
10. $\frac{8}{9}$ of 45 $=$ $1\frac{2}{3}$ times what number?

*Remark.* This question is equivalent to $\frac{8}{9}$ of 45 $=$ $\frac{5}{3}$ of what number?

11. $\frac{8}{7}$ of 49 $=$ $2\frac{2}{3}$ times what number?
12. $\frac{4}{9}$ of 72 $=$ $1\frac{4}{5}$ times what number?
13. $\frac{8}{7}$ of 28 $=$ $1\frac{2}{3}$ times what number?
14. $\frac{3}{7}$ of 63 $=$ $3\frac{1}{2}$ times what number?
15. $\frac{4}{5}$ of 45 $=$ $3\frac{1}{2}$ times what number?
16. $\frac{7}{8}$ of 48 $=$ $1\frac{1}{7}$ times what number?
17. *$3\frac{2}{5}$ times 10 $=$ $1\frac{3}{4}$ times what number?
18. $7\frac{3}{4}$ times 8 $=$ $4\frac{1}{2}$ times what number?
19. $8\frac{2}{3}$ times 9 $=$ $2\frac{2}{3}$ times what number?
20. $5\frac{1}{2}$ times 8 $=$ $1\frac{1}{4}$ times what number?
21. $9\frac{1}{3}$ times 6 $=$ $2\frac{3}{4}$ times what number?
22. $6\frac{5}{7}$ times 7 $=$ $1\frac{4}{5}$ times what number?
23. $8\frac{1}{2}$ times 6 $=$ $3\frac{1}{2}$ times what number?
24. $7\frac{1}{2}$ times $\frac{4}{9}$ $=$ $\frac{5}{7}$ of what number?

*Solution.* 7 times $\frac{4}{9}$ $=$ $\frac{28}{9}$, and $\frac{1}{2}$ of $\frac{4}{9}$ $=$ $\frac{2}{9}$, which added to $\frac{28}{9}$ $=$ $\frac{30}{9}$. If $\frac{30}{9}$ $=$ $\frac{5}{7}$ of some number, $\frac{1}{7}$ of that number must equal $\frac{1}{5}$ of $\frac{30}{9}$, which is $\frac{6}{9}$, and $\frac{7}{7}$ or the number must equal 7 times $\frac{6}{9}$, which are $\frac{42}{9}$, or $4\frac{6}{9}$. Therefore, $7\frac{1}{2}$ times $\frac{4}{9}$ $=$ $\frac{5}{7}$ of $4\frac{6}{9}$.

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* $3\frac{2}{5}$ should not be reduced to an improper fraction.

25. $8\frac{3}{4}$ times $\frac{8}{9}$ = $1\frac{2}{5}$ times what number?
26. $9\frac{4}{5}$ times $\frac{5}{8}$ = $1\frac{3}{4}$ times what number?
27. $3\frac{1}{2}$ times $\frac{8}{10}$ = $\frac{4}{5}$ times what number?
28. $2\frac{2}{3}$ times $\frac{9}{10}$ = $1\frac{1}{7}$ times what number?

**E.** 1. If $\frac{3}{4}$ of a gallon of molasses cost 25 cents, what will 1 gallon cost?

*Solution.* If $\frac{3}{4}$ of a gallon of molasses cost 25 cents, $\frac{1}{4}$ of a gallon will cost $\frac{1}{3}$ of 25 cents, which is $8\frac{1}{3}$ cents, and $\frac{4}{4}$ or 1 gallon will cost 4 times $8\frac{1}{3}$ cents, or $33\frac{1}{3}$ cents. Therefore, 1 gallon of molasses will cost $33\frac{1}{3}$ cents if $\frac{3}{4}$ of a gallon cost 25 cents.

2. If $\frac{5}{8}$ of a yard of muslin cost 37 cents, what will 1 yard cost?

3. If $\frac{6}{7}$ of a yard of linen cost 53 cents, what will 1 yard cost?

4. If $\frac{3}{5}$ of a month's wages amount to 23 dollars, what will 1 month's wages amount to?

5. John is 17 years old, and his age is $\frac{3}{8}$ of his teacher's. How old is his teacher?

6. Deborah says she has 41 cents. Then, says Lavinia, you have only $\frac{7}{10}$ as many as I have. How many cents did Lavinia have?

7. David told George that $\frac{6}{7}$ of his money would buy $5\frac{1}{3}$ pounds of raisins at 9 cents per pound. Then, replied George, you have 6 cents more than I have. How many cents had each of the boys?

8. Seth's father gave him a half dollar to buy a pound of tea with, saying to him, "$\frac{4}{5}$ of a pound of tea will cost 30 cents, and if you will tell me how much a pound will cost, you may have the money there will be left after paying for the tea." Seth answered correctly. What was his answer? How many cents did he have left after paying for the tea?

9. A farmer gave $7\frac{2}{3}$ dozen of eggs at 9 cents per dozen for $\frac{7}{8}$ of a gallon of oil. What was a gallon of the oil worth?

10. A butcher received $5\frac{5}{8}$ dollars for $\frac{7}{8}$ of a hundred weight of beef. What would he have received for a hundred weight at the same price? What would he have received for $\frac{9}{10}$ of a hundred weight?

11. A schoolmaster being asked his age replied, "$\frac{5}{8}$ of my life has been spent in teaching. I have taught in Boston 25 years, which is $\frac{5}{8}$ of all the time I have spent in teaching." What was his age?

12. In travelling on the rail-road from Springfield, we pass through Wilbraham and Spencer, and the distance from Springfield to Wilbraham is $\frac{1}{9}$ of the distance from Springfield to Worcester; the distance from Wilbraham to Spencer is $\frac{5}{9}$ of the distance from Springfield to Worcester, and from Spencer to Worcester it is 18 miles. How far is it from Springfield to Worcester? How far from Springfield to Wilbraham? from Wilbraham to Spencer? from Springfield to Spencer?

13. Marcellus spent $\frac{5}{9}$ of his money for paper, $\frac{2}{9}$ of it for pens, and then had 10 cents left, which he spent for ink. How much money had he at first? How many cents did he spend for paper? for pens?

14. If a yard of cambric cost 36 cents, and $\frac{5}{6}$ of a yard of cambric cost as much as $\frac{5}{7}$ of a yard of muslin, what will a yard of muslin cost?

15. If a yard of linen cost 56 cents and $\frac{6}{7}$ of a yard of linen cost $\frac{8}{9}$ as much as a yard of lawn, how much will a yard of lawn cost? What will $\frac{7}{9}$ of a yard of lawn cost?

16. If Joseph can earn 54 cents per day, and if it takes William $\frac{3}{10}$ of a day to earn as much as Joseph can earn in $\frac{4}{9}$ of a day, how much can William earn in one day? in $\frac{5}{8}$ of a day?

17. If a yard of muslin cost 70 cents, and $\frac{3}{7}$ of a yard of muslin cost $\frac{5}{6}$ as much as a yard of cambric, what will a yard of cambric cost? What will $\frac{3}{7}$ of a yard of cambric cost?

18. A farmer, after selling his crops, found that he had received 72 dollars for hay, and that $\frac{5}{9}$ of what he had received for hay was 2 dollars less than $\frac{7}{9}$ of what he had received for potatoes. How many dollars had he received for potatoes? $\frac{5}{6}$ of what he had received for potatoes was 4 dollars more than $\frac{4}{9}$ of what he had received for corn. How many dollars had he received for corn?

19. A farmer, after selling his crops, found that he had received 81 dollars for corn, and that $\frac{4}{9}$ of what he had received for corn was 4 dollars less than $\frac{5}{9}$ of what he had received for potatoes; and that $\frac{7}{9}$ of what he received for potatoes was 2 dollars more than $\frac{6}{9}$ of what he had received for hay. How many dollars had he received for hay?

20. Abner has $5\frac{3}{8}$ dollars, and $\frac{5}{8}$ of what he has is $\frac{3}{8}$ of a dollar more than $\frac{4}{7}$ of what his brother Granville has. How many dollars has his brother Granville?

21. Granville has $6\frac{3}{8}$ dollars, and $\frac{4}{7}$ of what he has is $\frac{3}{8}$ of a dollar more than $\frac{5}{8}$ of what his brother Abner has. How many dollars has Abner?

22. Mr. Battles gave 25 dollars for a cow. $1\frac{4}{5}$ times what he gave for the cow is equal to $2\frac{1}{4}$ times what he gave for a heifer. What did he give for the heifer?

23. Sidney gathered 64 quarts of walnuts. $\frac{1}{8}$ of what he gathered was $\frac{1}{5}$ of what Alfred gathered. $\frac{3}{4}$ of what Alfred gathered was 6 quarts less than $\frac{4}{5}$ of what Howard gathered. $1\frac{1}{9}$ times what Howard gathered was $\frac{5}{7}$ of what Oliver gathered. How many quarts did Oliver gather?

24. Oliver gathered 70 quarts of walnuts. $\frac{5}{7}$ of what he gathered was $1\frac{1}{9}$ times what Howard gathered. $\frac{4}{5}$ of what Howard gathered was 6 quarts more than $\frac{3}{4}$ of what Alfred gathered. $\frac{1}{5}$ of what Alfred gathered was $\frac{1}{8}$ of what Sidney gathered. How many quarts did Sidney gather?

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## SECTION XV.

**A.** 1. 3 times 2 tens = how many tens? How many units? Then, how many are 3 times 20? 20 times 3?

2. 8 times 4 tens = how many tens? How many units? Then, how many are 8 times 40? 40 times 8?

Let the pupil now read the following, supplying appropriate numbers in place of the stars.

3. 7 times 6 tens = * tens, or * units; then 7 times 60 = *.
4. 9 times 9 tens = * tens, or * units; then 9 times 90 = *.
5. 4 times 7 tens = * tens, or * units; then 4 times 70 = *.
6. 8 times 8 tens = * tens, or * units; then 8 times 80 = *.
7. 3 times 9 tens = * tens, or * units; then 3 times 90 = *.
8. 6 times 4 tens = * tens, or * units; then 6 times 40 = *.
9. 7 times 8 tens = * tens, or * units; then 7 times 80 = *.
10. 4 times 9 tens = * tens, or * units; then 4 times 90 = *.
11. 6 times 5 tens = * tens, or * units; then 6 times 50 = *.
12. 5 times 6 tens = * tens, or * units; then 5 times 60 = *.

**B.** 1. How many are $4 \times 3$? $4 \times 30$? $40 \times 3$? $3 \times 40$? $30 \times 4$?

2. How many are $7 \times 3$? $7 \times 30$? $70 \times 3$? $30 \times 7$? $3 \times 70$?

3. How many are 6 × 8? 6 × 80? 60 × 8? 80 × 6? 8 × 60?

4. How many are 9 × 5? 90 × 5? 9 × 50? 5 × 90?

5. How many are 8 × 7? 8 × 70? 7 × 80? 70 × 8?

6. How many are 5 × 6? 50 × 6? 60 × 5? 6 × 50?

7. How many are 9 × 9? 90 × 9? 9 × 90?

8. How many are 4 × 7? 40 × 7? 4 × 70? 7 × 40?

9. How many are 8 × 6? 8 × 60? 80 × 6? 60 × 8?

10. How many are 7 × 9? 9 × 70? 70 × 9? 7 × 90?

**C.** 1. How many times 7 tens = 21 tens? Then, how many times 70 = 210?

2. How many times 5 tens = 25 tens? Then, how many times 50 = 250?

3. How many times 6 tens = 42 tens? Then, how many times 60 = 420?

4. How many times 8 tens = 40 tens? Then, how many times 80 = 400?

5. How many times 4 tens = 12 tens? Then, how many times 40 = 120?

Supply appropriate numbers, in place of the stars, in the following examples:

6. 81 tens = 9 times 9 tens; therefore, 810 = 9 times 90.

7. 18 tens = * times 2 tens; therefore, 180 = * times 20.

8. 27 tens = * times 3 tens; therefore, 270 = * times 30.

9. 90 tens = * times 9 tens; therefore, 900 = * times 90.

10. 54 tens = * times 6 tens; therefore, 540 = * times 60.

11. 63 = 9 times 7; therefore, 630 = 9 times 70 = 90 times 7.

12. 45 = * times 9; therefore, 450 = * times 90 = * times 9.

13. 45 = * times 5; therefore, 450 = * times 50 = * times 5.

14. 32 = * times 4; therefore, 320 = * times 4 = * times 40.

15. 70 = * times 7; therefore, 700 = * times 7 = * times 70.

16. 30 = * times 6; therefore, 300 = * times 6 = * times 60.

17. 72 = * times 9; therefore, 720 = * times 90 = * times 9.

18. 72 = * times 8; therefore, 720 = * times 8 = * times 80.

19. $\frac{1}{4}$ of 36 = 9; therefore, $\frac{1}{4}$ of 360 = 90, and $\frac{1}{40}$ of 360 = 9.

20. $\frac{1}{9}$ of 36 = *; therefore, $\frac{1}{9}$ of 360 = *, and $\frac{1}{90}$ of 360 = *.

21. $\frac{1}{4}$ of 16 = *; therefore, $\frac{1}{4}$ of 160 = *, and $\frac{1}{40}$ of 160 = *.

22. $\frac{1}{7}$ of 49 = *; therefore, $\frac{1}{7}$ of 490 = *, and $\frac{1}{70}$ of 490 = *.

23. $\frac{1}{8}$ of 40 = *; therefore, $\frac{1}{80}$ of 400 = *, and $\frac{1}{8}$ of 400 = *.

24. $\frac{1}{6}$ of 54 = *; therefore, $\frac{1}{60}$ of 540 = *, and $\frac{1}{6}$ of 540 = *.

25. $\frac{1}{5}$ of 35 = *; therefore, $\frac{1}{50}$ of 350 = *, and $\frac{1}{5}$ of 350 = *.

26. $\frac{1}{3}$ of 27 = *; therefore, $\frac{1}{30}$ of 270 = *, and $\frac{1}{3}$ of 270 = *.

**D.** 1. 6 = how many times 2? 1? 3? 6? Then, 60 = how many times 20? 10? 3? 6?

2. 24 = how many times 8? 6? 3? 4? Then, 240 = how many times 80? 8? 60? 6? 3? 30? 4? 40?

3. 30 = how many times 6? 10? 3? 5? Then, 300 = how many times 6? 10? 3? 5? 60? 1? 30? 50?

4. 420 = how many times 7? 6? 70? 60?

5. 630 = how many times 90? 7? 70? 9?

6. 720 = how many times 80? 90? 8? 9?

7. 400 = how many times 4? 40? 8? 100? 5? 50? 80? 10?

8. 180 = how many times 2? 90? 6? 3? 20? 9? 60? 30?

9. 480 = how many times 6? 60? 80? 8?

10. 540 = how many times 90? 60? 9? 6?
11. 350 = how many times 50? 70? 5? 7?
12. 560 = how many times 7? 8? 80? 70?

**E.** 1. What is $\frac{1}{7}$ of 280? of 420? of 630? of 560? of 700?
2. What is $\frac{1}{70}$ of 280? of 420? of 630? of 560? of 700?
3. What is $\frac{1}{9}$ of 630? of 360? of 810? of 720? of 450?
4. What is $\frac{1}{90}$ of 630? of 360? of 810? of 720? of 450?
5. What is $\frac{1}{5}$ of 150? of 450? of 300? of 200? of 250?
6. What is $\frac{1}{50}$ of 150? of 450? of 300? of 200? of 250?
7. What is $\frac{1}{60}$ of 240? of 420? of 600? of 540? of 300?
8. What is $\frac{1}{80}$ of 560? of 320? of 480? of 720? of 400?
9. What is $\frac{1}{40}$ of 320? of 400? of 200? of 160? of 360?
10. What is $\frac{1}{30}$ of 60? of 120? of 300? of 240? of 90?
11. What is $\frac{4}{9}$ of 540? $\frac{3}{5}$ of 300? $\frac{7}{90}$ of 540?
12. What is $\frac{3}{4}$ of 120? $\frac{5}{6}$ of 240? $\frac{3}{7}$ of 350?
13. What is $\frac{2}{5}$ of 450? $\frac{7}{9}$ of 810? $\frac{5}{8}$ of 400?
14. What is $\frac{7}{50}$ of 450? $\frac{2}{70}$ of 490? $\frac{5}{80}$ of 480?
15. What is $\frac{9}{40}$ of 360? $\frac{7}{30}$ of 180? $\frac{4}{7}$ of 630?

**F.** 1. 640 = $\frac{8}{9}$ of what number?
2. 450 = $\frac{9}{10}$ of what number?
3. 560 = $\frac{7}{9}$ of what number?
4. 360 = $\frac{4}{9}$ of what number?
5. 80 = $\frac{4}{5}$ of what number?
6. 320 = $\frac{4}{7}$ of what number?
7. 18 = $\frac{3}{20}$ of what number?
8. 35 = $\frac{7}{40}$ of what number?
9. 18 = $\frac{9}{50}$ of what number?
10. 48 = $\frac{6}{70}$ of what number?
11. 28 = $\frac{4}{30}$ of what number?
12. 21 = $\frac{7}{30}$ of what number?
13. 80 = $6\frac{4}{6}$ times what number?
14. 400 = $8\frac{8}{9}$ times what number?
15. 240 = $8\frac{4}{7}$ times what number?
16. 420 = $8\frac{6}{8}$ times what number?
17. 540 = $1\frac{4}{5}$ times what number?

18. 180 = $1\frac{1}{5}$ times what number?
19. 180 = $4\frac{1}{2}$ times what number?
20. 640 = $2\frac{2}{3}$ times what number?
21. 350 = $5\frac{5}{6}$ times what number?
22. 210 = $4\frac{2}{7}$ times what number?

**G.** 1. 9 × 20 + 60, ÷ 40, × 5, × 7 + 90, ÷ 5, ÷ 3, ÷ 4, × 8, × 7, + 40 = how many times 8?

2. 30 + 90 + 60 + 50 + 40, ÷ 9 + 330, ÷ 4, × 7 + 10, ÷ 80, × 70 — 20 = how many times 6?

3. 810 ÷ 9 — 30, × 6, + 9 × 40, ÷ 80, + 9 × 9 + 90, ÷ 30, × 50 = how many times 30?

4. 5 × 70 + 50 + 80, ÷ 6, ÷ 4, × 7 + 3 × 30 + 310, ÷ 6, ÷ 30, × 10, × 6 = how many times 90?

5. 10 × 7 × 7 + 70, ÷ 8 — 20, × 5 + 70, ÷ 40, × 5, × 9, ÷ 6, ÷ 3, × 9 = how many times 9?

6. 10 × 3 × 3 × 3 + 30, ÷ 60, × 6, × 6, ÷ 2, × 7 + 70, ÷ 70, × 5, × 5, — 40 = how many times 70?

7. $\frac{3}{4}$ of 80 × $\frac{1}{40}$ of 360 + $\frac{1}{7}$ of 140, ÷ $\frac{1}{7}$ of 490, × $\frac{2}{3}$ of 360 — $\frac{2}{3}$ of 120 = how many times 8?

8. $\frac{7}{8}$ of 720 + $\frac{2}{50}$ of 250, ÷ $\frac{2}{7}$ of 280, × $\frac{3}{7}$ of 210 + $\frac{4}{5}$ of 100 = how many times 80?

9. $\frac{2}{7}$ of 280 × $\frac{3}{50}$ of 150 — 8 × 60, ÷ $\frac{2}{7}$ of 210, × $\frac{4}{9}$ of 180 + 40 = how many times 60?

**H.** 1. I bought of Mr. Grover 3 lb. of cloves at 50 cents per lb., 4 lb. of cinnamon at 40 cents per lb., and 7 lb. of citron at 40 cents per pound. What was the amount of my purchase? Gave in payment 5 bushels of potatoes at 90 cents per bushel, and paid the rest in butter at 20 cents per lb. How many lb. of butter did I give?

2. Rode 8 hours at the rate of 20 miles per hour, 9 hours at the rate of 10 miles per hour, and 4 hours at the rate of 30 miles per hour. How many miles did I ride? On my return I rode 180 miles at the rate of 30 miles per hour, 50 miles at the rate of 10 miles per hour, and the remaining distance at the rate of 20 miles per hour. How many hours was I returning?

3. A farmer gave a trader 40 lb. of cheese at 9 cents per lb. for cloth at 60 cents per yard, 30 lb. of cheese at 8 cents per lb.

for rice at 6 cents per lb., and 4 dozen eggs at 20 cents per dozen, for sugar at 8 cents per lb. How many yards of cloth did the farmer receive? How many lb. of rice? How many lb. of sugar?

4. During the first week in February, Mr. Hamilton earned 60 cents per day; during the second he earned 90 cents per day, during the third he earned 70 cents per day, and during the fourth he earned 80 cents per day. How much did he earn in the month, there being 6 working days in a week?

5. A farmer exchanged 7½ acres of land worth 60 dollars per acre, for land worth 50 dollars per acre. How many acres did he receive?

6. Gave 8⅓ bushels of corn at 60 cents per bushel, for potatoes at 50 cents per bushel. How many bushels of potatoes did I receive?

7. Gave 9 yoke of oxen worth 70 dollars per yoke, and a calf worth 10 dollars, for ploughs at 8 dollars apiece. How many ploughs did I receive? Sold sixty of the ploughs at 9 dollars apiece and the rest at 8½ dollars apiece. What did I receive for them all? How much did I gain by the transaction?

8. If a bushel of potatoes weighs 40 lb., and a bushel of wheat weighs 60 lb., how much more will 8 bushels of wheat weigh than 9 bushels of potatoes?

9. A shoemaker gave 9 pairs of shoes worth 80 cents per pair, for 6 gallons of molasses at 30 cents per gallon, and sugar at 9 cents per pound. How many lb. of sugar did he receive?

10. A bushel of clover-seed weighs 60 lb., and 6⅔ bushels of clover-seed weigh as much as 8 bushels of salt. How much does a bushel of salt weigh?

11. A man bought 9⅐ gallons of brandy at 70 cents per gallon. He took out 1⅐ gallons for his own use, and then sold the rest for what the whole of it cost him. For what did he sell it per gallon?

12. Worked 6¾ days at 80 cents per day, and received in payment 6 yards of cloth. I sold 4 yards of the cloth at cost. How much did I get for it?

13. George is 20 years of age and his father is 3 times as old. How old is his father? 4 times George's age added to 8 times his father's age, is equal to 7 times the age of his Uncle William. How old is his Uncle William?

14. Mr. Crane and Mr. Snow were asked how old they were. Mr. Snow replied, "I am 20 years older than Mr. Crane, and 7½ times the difference between our ages is three times my age." How old was Mr. Snow? was Mr. Crane?

15. A lady being asked her age replied, "My father is 30 ars older than my sister Sarah, and 8⅓ times the difference tween their ages is 5 times my father's age. Now, if you will ll me how old my father and sister are, I will tell you how you n find my age." What were the ages of her father and sister? correct answer having been given the lady said, "To 3 times y father's age add 6 times my sister's age and you will obtain sum ⅓ of which will be 9 years more than 4½ times my age." Vhat was her age?

16. ⅚ of the value of a piece of cloth, worth 36 dollars, is 10 lollars more than all the money Horace has. How much money ias Horace?

17. The value of 50 barrels of flour at 8 dollars per barrel is 0 dollars less than 1⅙ times the cost of an acre of land. What vill be the cost of ¾ of an acre of land at the same rate?

18. A man in Albany bought a lot of flour. He sent 3/7 of it o N. York, 2/7 of it to Boston, and the rest of it, 180 barrels, he shipped to England. How many barrels of flour did he buy? How many did he send to N. York? to Boston?

19. If to 9⅓ times the number of states belonging to the United States in 1850, 80 be added, ⅓ of the result will be 27 more than the number of states originally belonging to the Union. Moreover, if 7 be added to the number of original states, 6 times the sum obtained will be 30 less than ⅜ of 400. How many states were there originally in the Union? How many in 1850?

## SECTION XVI.

**A.** 1. How many are 9 times 27?

*Solution.* $9 \times 27 = 9 \times 20 + 9 \times 7$; but $9 \times 20 = 180$, and $9 \times 7 = 63$; $180 + 63 = 243$. Therefore, $9 \times 27 = 243$.

NOTE. As soon as the principle is understood, the pupil should solve the problem by naming only the results. Thus, $9 \times 27 = 180 + 63 = 243$.

2. How many are $5 \times 49$? $7 \times 63$? $8 \times 96$? $4 \times 43$?
3. How many are $3 \times 99$? $2 \times 87$? $8 \times 37$? $5 \times 56$?
4. How many are $4 \times 87$? $3 \times 49$? $9 \times 28$? $3 \times 48$?
5. How many are $9 \times 87$? $5 \times 49$? $8 \times 69$? $6 \times 13$?

6. How many are $3 \times 54$? $2 \times 96$? $8 \times 14$? $9 \times 54$?

7. How many are 98 times 3?

*Solution.* $98 \times 3 = 90 \times 3 + 8 \times 3 = 270 + 24 = 294$.

8. How many are $59 \times 9$? $47 \times 4$? $83 \times 6$? $17 \times 7$?

9. How many are $57 \times 8$? $96 \times 6$? $97 \times 10$? $78 \times 8$?

10. How many are $85 \times 10$? $57 \times 7$? $63 \times 4$? $76 \times 8$?

11. How many are $75 \times 6$? $96 \times 9$? $66 \times 8$? $98 \times 7$?

**B.** 1. What is the greatest multiple of 6 that can be taken from the tens of 216, and what remainder will it leave if subtracted from 216?

*Ans.* The greatest multiple of 6 that can be taken from the tens of 216 is 18 tens, and if this be subtracted from 216, there will be a remainder of 36.

2. Into what parts then will you separate 216 for convenience of division by 6?

*Ans.* 180 and 36, because 180 is the greatest multiple of 6 that can be taken from the tens of 216.

3. $216 =$ how many times 6?

*Solution.* $216 \div 6 = 180 \div 6 + 36 \div 6 = 30 + 6 = 36$.

Or, by *Solution* 2nd, 6 is contained in 21 tens 3 tens times, with a remainder of 3 tens. But 3 tens = 30 units, which added to the 6 units equals 36 units. 6 is contained in 36, 6 times. Therefore, 216 equals 36 times 6.

NOTE. Both forms of solution should be dispensed with as soon as the pupil can readily give the results without them.

4. What is the greatest multiple of 7 that can be taken from the tens of 245, and what remainder will it leave if subtracted from 245?

5. Into what parts, then, will you separate 245 for convenience of division by 7?

6. $245 =$ how many times 7?

7. Into what parts will you separate 261 for convenience of division by 9?

8. $261 =$ how many times 9?

9. Into what parts will you separate 609 for convenience of division by 7? by 9? by 10? by 8?

10. 609 = how many times 7? 9? 10? 8?

11. Into what parts will you separate 456 for convenience of division by 8? by 5? by 10? by 7? by 6?

12. 456 = how many times 8? 5? 10? 7? 6?

13. Into what parts will you separate 532 for convenience of division by 8? by 6? by 10? by 9? by 7?

14. 532 = how many times 8? 6? 10? 9? 7?

15. 117 = how many times 9? 6? 2? 3? 10? 4? 5? 7?

16. 219 = how many times 7? 3? 5? 10? 8? 6? 4? 9?

17. 324 = how many times 4? 8? 9? 10? 6? 7? 5?

18. 168 = how many times 2? 5? 7? 9? 4? 3? 10? 6? 8?

19. 147 = how many times 5? 9? 4? 6? 3? 7? 2? 8? 10?

20. 288 = how many times 4? 10? 8? 7? 3? 6? 9? 5?

21. 93 = how many times 9? 7? 4? 6? 2? 3? 8? 7? 10?

22. 427 = how many times 8? 9? 10? 6? 7? 5?

23. 224 = how many times 3? 9? 8? 4? 5? 7? 6? 10?

24. 100 = how many times 5? 6? 4? 3? 7? 8? 2? 10? 9?

25. 316 = how many times 10? 4? 6? 9? 7? 5? 8?

26. 237 = how many times 9? 4? 8? 3? 6? 7? 10? 5?

27. 627 = how many times 6? 8? 9? 10? 7?

28. 76 = how many times 7? 8? 9? 4? 3? 5? 2? 10? 6?

**C.** 1. What is $\frac{1}{7}$ of 459?

*Solution.* $\frac{1}{7}$ of 459 = $\frac{1}{7}$ of 420 + $\frac{1}{7}$ of 39 = 60 + $5\frac{4}{7}$ = $65\frac{4}{7}$.

2. What is $\frac{1}{5}$ of 437? $\frac{1}{6}$ of 532? $\frac{1}{9}$ of 837? $\frac{1}{7}$ of 96?

3. What is $\frac{1}{8}$ of 137? $\frac{1}{9}$ of 561? $\frac{1}{7}$ of 628? $\frac{1}{2}$ of 37?

4. What is $\frac{3}{4}$ of 75? $\frac{2}{3}$ of 73? $\frac{5}{9}$ of 94? $\frac{8}{9}$ of 736?

5. What is $\frac{2}{3}$ of 58? $\frac{4}{5}$ of 83? $\frac{7}{9}$ of 279? $\frac{7}{10}$ of 537?

6. What is $\frac{2}{3}$ of 217? $\frac{3}{7}$ of 96? $\frac{3}{4}$ of 99? $\frac{5}{6}$ of 100?

7. What is $\frac{5}{9}$ of 137? $\frac{1}{2}$ of 175? $\frac{8}{9}$ of 671? $\frac{3}{10}$ of 981?

8. What is $\frac{3}{8}$ of 577? $\frac{6}{7}$ of 666? $\frac{2}{3}$ of 254? $\frac{5}{8}$ of 261?

9. What is $\frac{2}{3}$ of 289? $\frac{3}{7}$ of 645? $\frac{3}{10}$ of 741? $\frac{8}{9}$ of 423?

10. What is $\frac{1}{2}$ of 89? $\frac{1}{3}$ of 79? $\frac{6}{7}$ of 459? $\frac{1}{4}$ of 87?

**D.** 1. 167 is $\frac{3}{5}$ of what number?

*Solution.* If 167 is $\frac{3}{5}$ of some number, $\frac{1}{5}$ of that number must be $\frac{1}{3}$ of 167, which is $55\frac{2}{3}$, and $\frac{5}{5}$ or the number must be 5 times $55\frac{2}{3}$. 5 times 55 are 275, and 5 times $\frac{2}{3}$ are $\frac{10}{3}$, or $3\frac{1}{3}$, which added to 275 are $278\frac{1}{3}$. Therefore, $167 = \frac{3}{5}$ of $278\frac{1}{3}$.

2. $229 = \frac{7}{8}$ of what number?
3. $621 = \frac{9}{10}$ of what number?
4. $81 = \frac{5}{9}$ of what number?
5. $254 = \frac{5}{6}$ of what number?
6. $281 = \frac{3}{7}$ of what number?
7. $329 = \frac{7}{8}$ of what number?
8. $117 = \frac{3}{4}$ of what number?
9. $37 = \frac{3}{5}$ of what number?
10. $71 = \frac{4}{7}$ of what number?
11. $311 = \frac{5}{8}$ of what number?
12. $87 = \frac{5}{9}$ of what number?
13. $358 = 2\frac{2}{3}$ times what number?
14. $418 = 1\frac{4}{5}$ times what number?
15. $615 = 2\frac{1}{3}$ times what number?
16. $93 = 1\frac{3}{4}$ times what number?
17. $834 = 4\frac{1}{2}$ times what number?

**E.** 1. How many are 9 times 268?

*Solution.* $9 \times 268 = 9 \times 200 + 9 \times 60 + 9 \times 8 = 1800 + 540 + 72 = 2412$.

2. How many are 7 × 327? 5 × 861? 9 × 468?
3. How many are 4 × 813? 2 × 917? 6 × 321?
4. How many are 8 × 612? 3 × 999? 5 × 875?
5. How many are 6 × 348? 8 × 527? 9 × 813?
6. How many are 8 × 465? 2 × 981? 7 × 387?
7. How many are 8 × 837? 4 × 958? 6 × 372?
8. How many are 3 × 119? 4 × 549? 4 × 591?
9. How many are 4 × 217? 6 × 459? 9 × 645?
10. How many are 4 × 279? 4 × 729? 4 × 927?
11. How many are 9 × 578? 9 × 758? 9 × 875?

**F.** 1. 1396 = how many times 4?

*Solution.* $1396 \div 4 = 1200 \div 4 + 160 \div 4 + 36 \div 4 = 300 + 40 + 9 = 349$.

*Another Solution.* 4 is contained in 1200, 300 times, in 160, 40 times, and in 36, 9 times. Therefore, 1396 = 349 times 4.

*Another Solution.* 4 is contained in 1300, 400 times with a remainder of 1 hundred, or 10 tens, which added to 9 tens make 19 tens. 4 is contained in 19 tens, 4 tens times with a remainder of 3 tens, or 30 units, which added to 6 units are 36 units. 4 is contained in 36, 9 times. Therefore, 1396 = 349 times 4.

NOTE. The only object of the above forms of solution is to explain the nature of the process; when that is well understood, the explanation should be dropped, and results only named. Thus, in performing the above division, the pupil need only say, "4 is contained in 1396, 3 hundreds 4 tens and 9 units times, or 349 times." After a little practice in this way, the final result may be named at once. Thus, "1396 = 349 times 4."

2. 1404 = how many times 9? 4? 3? 6? 8? 7? 5? 2?
3. 2826 = how many times 3? 6? 9? 8? 7? 10? 2? 5?
4. 2873 = how many times 5? 8? 9? 3? 7? 4? 6? 10?
5. 1328 = how many times 6? 9? 3? 10? 7? 4? 8? 5?
6. 3418 = how many times 8? 9? 3? 7? 5? 2? 6? 4? 10?
7. 4176 = how many times 4? 6? 8? 2? 3? 5? 7? 9? 10?

**G.** 1. 2 cwt. 1 qr. 23 lb. = how many pounds?

*Solution.* Since 1 cwt. = 4 qr., 2 cwt. must equal 2 times 4 qr., or 8 qr., to which adding 1 qr., we have 9 qr. as the value of 2 cwt. 1 qr. Since 1 qr. = 28 lb., 9 qr. must equal 9 times 28 lb., or 252 lb., to which adding 23 lb., we have 275 lb. Therefore, 2 cwt. 1 qr. 23 lb. = 275 lb.

2. 3 dwt. 16 gr. = how many grains?
3. 7 da. 19 h. = how many hours?
4. 1 cwt. 3 qr. 17 lb. = how many pounds?
5. 6s. 8d. 2qr. = how many farthings?
6. 6 lb. 4 oz. = how many drams?
7. 2 ℥. 2 ʒ. 2 ℈. 15 gr. = how many grains?
8. 1 sq. yd. 36 sq. in. = how many square inches?
9. 9 cu. yd. 18 cu. ft. = how many cubic feet?

10. 275 lb. = how many cwt. qr. and lb.?

*Solution.* Since 28 lb. = 1 qr., 275 lb. must equal as many quarters as there are times 28 lb. in 275 lb., which are 9 times with a remainder of 23 lb. But since 4 qr. = 1 cwt., 9 qr. must equal as many cwt. as there are times 4 qr. in 9 qr., which are 2 times with a remainder of 1 qr. Therefore, 275 lb. = 2 cwt. 1 qr. 23 lb.

11. 88 gr. = how many dwt. and gr.?
12. 213 lb. = how many lb. qr. and cwt.?
13. 187 hours = how many hours and days?
14. 322 qr. = how many s. d. and qr.?
15. 1600 drams = how many lb. and oz.?
16. 175 gr. = how many ʒ, ℈, and gr.?
17. 1332 sq. in. = how many sq. yd. and sq. in.?
18. 311 qt. = how many bu. pk. and qt.?
19. 255 gills = how many gal. qt. pt. and gi.?
20. 958 gr. = how many ℥, ʒ, ℈ and gr.?

**II.** 1. Sold 7 acres of land at 48 dollars per acre, and received in payment 9 cows at 27 dollars apiece, and the rest in money. How much money did I receive? I invested the money thus received in apples at 3 dollars per barrel. How many barrels did I buy?

2. Gave 9 lots of pasture land, each lot containing 87 acres, for 5 lots of tillage, each lot containing 64 acres. How many acres of land did I give? How many did I receive? How many more did I give than receive?

3. A drover bought 7 oxen for 47 dollars apiece; he sold them for 413 dollars. How many dollars did he get for each ox? How many dollars did he gain on each? How many dollars did he gain on all of them? He spent 8 dollars of the money he received for the oxen, and gave the rest of it for 9 cows, but being unfortunate he was obliged to sell the cows for 31 dollars apiece. How many dollars did he lose on each cow? How many on all? How many dollars did he receive for all the cows?

4. A man gave ⅜ of 672 dollars for 4 diamond rings. What would 9 rings cost at the same rate?

5. Joseph has 5 pieces of rope all of the same length, and Samuel has 1 piece 336 feet in length. By measuring they find

that $\frac{6}{7}$ of Samuel's rope is just equal in length to 2 of Joseph's pieces. How many feet of rope are there in all of Joseph's pieces?

6. A man bought a quantity of sugar. He sold 378 pounds of it, and then found that $\frac{2}{9}$ of what he had sold was equal to $\frac{7}{8}$ of what remained unsold. How much remained unsold?

7. Gave 9 casks of raisins, each cask containing 96 lb., for 8 cwt. of dates. How many lb. of raisins would 7 cwt. of dates be worth at the same rate?

8. Edmund had a little basket that held just 76 chestnuts. He lent it to Ralph and Edgar, on condition that they should give him chestnuts enough to fill it 6 times to pay for the use of it. It turned out that Ralph used the basket just 3 times as much as Edgar. How many chestnuts ought Edgar in justice to pay? Ought Ralph?

9. A man gave $\frac{3}{4}$ of 432 dollars for $\frac{6}{7}$ of an acre of land. What would an acre have cost? $\frac{5}{9}$ of an acre of this land is worth only $\frac{3}{10}$ of what the same man gave for a fast horse. How much did he give for his fast horse? What he gave for the fast horse is 20 dollars less than $\frac{8}{9}$ of the money he owes. How much money does he owe?

10. Bought $7\frac{3}{4}$ bags of coffee, each bag containing 45 lb. and $5\frac{2}{3}$ bags, each bag containing 57 lb. How many pounds of coffee did I buy in all?

11. Bought $3\frac{7}{8}$ pieces of cloth each piece containing 96 yards, $4\frac{3}{5}$ pieces each piece containing 75 yards, and $2\frac{5}{7}$ pieces each piece containing 91 yards. How many yards did I buy?

12. Gave $7\frac{5}{12}$ bags of corn at 96 cents per bag, for 8 bushels of wheat. How many cents would 7 bushels of wheat cost at the same rate?

13. Seth put 48 bushels of wheat into bags, each bag containing 3 pecks. How many bags did it take?

14. John put 20 lb. 12 oz. of candy into 4 oz. packages. How many packages did it take?

15. Abner put 30 lb. 12 oz. of mustard into 6 oz. canisters. How many canisters did it fill?

16. How many times must a chain 4 rods in length be laid down to measure a distance of 7 furlongs, 32 rods?

17. If to $\frac{5}{7}$ of all the money Ellery has, 24 dollars be added, the sum will be 114 dollars. How much money has he?

18. A man being asked the distance from Albany to Syracuse, replied, "It is 109 miles from Albany to Rome, and the distance to Rome is 2 miles less than $\frac{3}{4}$ of the distance to Syracuse." How far is it from Albany to Syracuse?

19. On the Ohio river it is 183 miles from Pittsburg to Marietta, and from Pittsburg to Wheeling it is 28 miles less than $\frac{2}{3}$ of this distance. How far is Wheeling from Pittsburg? from Marietta?

20. From Pittsburg to Marysville it is 426 miles, and $1\frac{1}{6}$ times this distance is 12 miles more than the distance from Pittsburg to Cincinnati. How far is it from Pittsburg to Cincinnati? from Cincinnati to Marysville?

21. It is 630 miles from Pittsburg to New Albany, and $\frac{2}{9}$ of this distance is 69 miles less than $\frac{1}{3}$ of the distance from Pittsburg to Louisville. How far is it from Pittsburg to Louisville?

22. $\frac{43}{70}$ of the distance from Pittsburg to New Albany is 57 miles more than $\frac{3}{8}$ of the distance from Pittsburg to the mouth of the Wabash. How far is it from Pittsburg to the mouth of the Wabash?

23. Add $\frac{1}{2}$ of the distance from Pittsburg to New Albany to $\frac{1}{3}$ of the distance from Pittsburg to Marysville, and the sum will be 19 miles more than $\frac{3}{7}$ of the distance from Pittsburg to Cairo, at the mouth of the Ohio. How far is Cairo from Pittsburg? from Marysville?

24. Wheeling is 94 miles from Pittsburg, and 28 miles less than $\frac{2}{3}$ of the distance of Marietta from Pittsburg. How far is Marietta from Pittsburg? from Wheeling?

25. Cincinnati, which is 485 miles from Pittsburg, is 12 miles less than $1\frac{1}{6}$ times the distance of Marysville from Pittsburg. How far is Marysville from Pittsburg? from Cincinnati?

26. Louisville is 627 miles from Pittsburg, and if 69 miles be subtracted from $\frac{1}{3}$ of this distance, the remainder will be $\frac{2}{9}$ of the distance of New Albany from Pittsburg. How far is New Albany from Pittsburg?

27. The mouth of the Wabash river is 880 miles from Pittsburg, and $\frac{3}{8}$ of this distance is 57 miles less than $\frac{43}{70}$ of the distance from Pittsburg to New Albany. What is the distance from New Albany to Pittsburg according to this statement? What is the distance from New Albany to the mouth of the Wabash?

28. Cairo is 1022 miles from Pittsburg. Prove, by using results found in the 26th and 27th examples, that $\frac{3}{7}$ of this distance is 19 miles less than the sum obtained by adding $\frac{1}{2}$ of the distance from Pittsburg to New Albany, to $\frac{1}{3}$ of the distance from Pittsburg to Marysville.

29. What do the last five examples show concerning the answers you obtained to the 19th, 20th, 21st, 22d, and 23d examples?

# SECTION XVII.

**A.** 1. A power of a number is the product obtained by taking the given number any number of times as a factor. It is sometimes defined as the product that would be obtained by multiplying the number once or more times by itself.

2. If a number is taken twice as a factor, it is said to be raised to the second power; if taken 3 times, to the third power, &c., &c. Thus, 9 = the second power of 3, because it is the product of 3 × 3, or 3 taken twice as a factor. 27 = the third power of 3, because it is the product of 3 × 3 × 3, or of 3 threes multiplied together, or of 3 taken 3 times as a factor.

3. The second power of a number is often called its square, and the third power its cube.

4. The process of finding the second power of a number is called squaring the number, or raising it to the second power.

5. The process of finding the third power of a number is called cubing the number, or raising it to the third power.

6. The process of finding the fourth power of a number is called raising it to the fourth power, — of finding the fifth power is called raising it to the fifth power, &c.

7. The power to which a number is to be raised, is often indicated by a small figure placed to the right and a little above the number to be raised. The figure so placed is called an exponent.

8. $5^2$ indicates that 5 is to be raised to the second power, *i. e.*, taken twice as a factor. Therefore $5^2 = 5 \times 5 = 25$.

9. $2^5$ indicates that 2 is to be raised to the fifth power, *i. e.*, taken 5 times as a factor. Therefore, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

10. $5^2$ is read *five square*, or *five second power*.

11. $2^5$ is read *two fifth power*.

12. The number thus repeated as a factor is called the root of the product obtained. Thus, 3 is the square root or second root of 9, because, taken as a factor twice, it will produce 9. It is the cube root or third root of 27, because, taken as a factor 3 times, it will produce 27. It is the fourth root of 81, because taken as a factor 4 times it will produce 81.

13. In like manner 2 is the square root or second root of 4, the cube root or third root of 8, the fourth root of 16, the fifth root of 32, the sixth root of 64, &c., &c.

**B.** 1. What is the square of 2? of 1? of 10? of 7? of 4?

2. What is the second power of 3? of 5? of 9? of 6? of 8?

3. What is the cube of 5?

*Solution.* The cube of 5 is the product of 5 taken as a factor 3 times, or of $5 \times 5 \times 5$; but $5 \times 5 = 25$, and $25 \times 5 = 125$. Therefore $5^3 = 125$.

4. What is the cube of 7? of 3? of 10? of 2?

5. What is the third power of 8? of 4? of 1? of 6? of 9?

6. What is the value of $5^4$? of $2^5$? of $7^3$? of $3^4$? of $4^5$? of $3^5$?

7. What is the square root of 81?

*Ans.* The square root of $81 = 9$, because 9 taken as a factor twice, or $9 \times 9$, equals 81.

8. What is the square root of 64? of 1? of 4? of 25?

9. What is the square root of 49? of 9? of 36? of 16?

10. What is the square root of 81? of 100?

11. What is the cube root of 8? of 729? of 1? of 343? of 64?

12. What is the cube root of 27? of 512? of 125? of 216? of 1000?

13. What is the fourth root of 81? of 256? of 16? of 625?

**C.** 1. How many square rods in a rectangular play-ground 9 rods long and 7 wide?

*Solution.* Since a space 1 rod long and 1 rod wide contains 1 square rod, a space 9 rods long and 1 rod wide must contain 9 square rods, and a space 9 rods long and 7 rods wide must contain 7 times 9 square rods, or 63 square rods. Therefore, a play-ground 9 rods long and 7 rods wide contains 63 square rods.

2. How many square rods in a field 40 rods long and 9 rods wide?

3. How many square feet in a platform 16 feet long and 9 feet wide? How many square yards?

4. How many square feet in a wall 98 feet long and 5 feet high? How many square yards?

5. How many square inches in a slate 10 inches long and 8 inches wide?

6. How many square yards in the floor of a room 6 yards long and $5\frac{1}{2}$ yards wide? How many square feet in the same floor?

7. How many square feet in a blackboard 16 feet long and $4\frac{1}{2}$ feet wide? How many square yards in the same board?

8. At 3 cents per square foot, what will it cost to paint a surface 16 feet long and 9 feet wide?

9. At 8 cents per square yard, what will it cost to plaster the ceiling of a school-room 14 yards long and 8 yards wide?

10. A teacher, wishing to obtain a blackboard 15 feet long and 6 feet wide, bought boards for the purpose, at 2$\frac{1}{2}$ cents per square foot. He hired a carpenter to make it, paying him 75 cents for his work. He paid 11 cents per square yard to have it painted and varnished, and it cost him 25 cents to have it brought to his school-room and put up. What was the whole cost of the board?

11. If a piece of board 14 feet long and 1$\frac{3}{7}$ feet wide costs 40 cents, how much would boards enough to cover a floor 20 feet long and 16 feet wide cost?

12. Isabella had a rectangular flower-bed of which she wished to find the contents. She therefore took a rule 1$\frac{1}{2}$ feet long, and, by measurement, found that the side of the flower-bed was 8, and the end 3 times as long as the rule. How many square feet did her flower-bed contain?

13. Hannah had a rectangular flower-bed which she measured with a stick 1$\frac{1}{3}$ feet long. She found the side to be 6, and the end 4 times as long as the stick. How many square feet did her flower-bed contain?

14. Martha had a flower-bed which she measured with a string 25 feet long. She found its side to be $\frac{2}{5}$, and its end to be $\frac{3}{10}$ as long as the string. How many square feet did her flower-bed contain?

15. Myra had a shawl which was 7 feet long and 6$\frac{1}{2}$ feet wide, and Ada had one 7$\frac{1}{2}$ feet long and 6 feet wide. Which contained the most square feet, and how many the most?

16. How many square feet would there be in the outer surface of a box 7 feet long, 5 feet wide, and 4 feet high? How much would it cost to paint it at 1$\frac{3}{4}$ cents per foot?

*Suggestions.* The top and bottom of the box will each be 7 feet long and 5 feet wide; the two sides will each be 7 feet long and 4 feet high; and the two ends will each be 5 feet long and 4 feet high. The sum of all these surfaces is the quantity required. Considerations like the following will usually very materially shorten the work in such examples. Since the box is 7 feet long and 5 feet wide, the distance round it must be $7 + 5 + 7 + 5$ feet $= 24$ feet. The sides and ends of the box, then, will be equivalent to a surface 24 feet long and 4 feet high. The top and bottom, each being 7 feet long and 5 feet wide, will together be equal to a surface 7 feet long and 10 feet wide.

17. How many feet of canvass would be required to cover 3 boxes, each box being 4 feet long, 2½ feet wide, and 2 feet high? What would be the cost of the canvass at 2½ cents per square foot?

18. A man bought a yard 12 feet long and 8 feet wide for 7 cents per square foot, and built around it a tight board fence 4 feet high. What did the land cost? What did the boards for the fence cost at 1⅜ cents per foot?

19. How many square feet in the walls of an enclosure 29 feet long and 12 feet wide, the walls being 9 feet high, and what would it cost to paint them at 8¾ cents per square yard?

20. If a lot of land is 8 rods in length, what must be its width that it many contain 1 rood?

*Solution.* If it were 8 rods long and 1 rod wide, it would contain 8 square rods. Therefore, to contain 1 rood or 40 square rods, it must be as many rods wide as there are times 8 in 40, which are 5 times. Therefore, it must be 5 rods wide.

21. If a rectangular township containing 56 square miles is 8 miles in length, what must be its width?

22. Mr. Shepard owns a square garden containing 81 square rods. What is the length of its side?

23. A certain township in the form of a square contains 36 square miles. What is the length of its side?

24. Jason had a rectangular piece of gold leaf containing 64 square inches, and on measuring it he found that its length and breadth were equal. What was its length?

25. My sitting-room and parlor are each 5 yards wide, but my parlor is 2 yards longer than my sitting-room. The floor of my sitting-room contains 30 square yards. What is the length of my parlor? How many square yards are there in my parlor floor?

26. My house-lot is 12 rods long, but if it were only 8 rods long, it would contain 40 rods less than it now does. How much does it contain?

27. Mr. Wiswall's house-lot is 11 rods long and 9 rods wide. Mr. Messenger's contains 21 rods more than Mr. Wiswall's, and is 12 rods long. How wide is it?

28. Mr. Hitch's garden is 16 rods long, but if it were 7 rods longer it would contain 63 more square rods. How wide is it? How many square rods does it contain?

29. Mr. Wade owns a square garden, and he finds that if he doubles its length without altering its width, it will contain 72 square rods. How long is the garden?

30. Mr. Jenks says that if his black-board were 2 ft. wider than it now is, it would contain 26 more square feet, but if it were 2

ft. longer it would contain 11 more square feet. What is its length and width, and how many square feet does it contain?

31. How many cubic feet in a block 9 ft. long, 8 ft. wide, and 7 ft. high?

*Solution.* Since the block is 9 ft. long and 8 ft. wide, there must be 9 times 8, or 72, sq. ft. in the bottom of it. But each foot in height will give 1 cubic foot for every square foot in the bottom; and as in this case there are 72 sq. ft. in the bottom, there must be 72 solid feet for each foot in height. Therefore, 7 ft. in height must give 7 times 72 cubic feet, or 504 cubic ft. Therefore, a block 9 ft. long, 8 ft. wide, and 7 ft. thick contains 504 cu. ft.

32. How many cubic inches in a brick 8 inches long, 4 inches wide, and 2 inches thick?

33. How many cubic feet in a box 16 ft. long, 4 ft. wide, and 3 ft. thick?

34. How many cubic feet in a pile of wood 32 ft. long, 4 ft. wide, and 3 ft. high? How many cord feet? How many cords?

35. How many cubic feet in a stick of timber 15 ft. long, 2 ft. wide, and $1\frac{2}{3}$ ft. thick?

36. Mr. Wales was offered his choice of 2 piles of wood, one of which was 14 ft. long, 8 ft. wide, and 4 ft. high, and the other was 16 ft. long, 6 ft. wide, and 4 ft. high. Not knowing how to calculate, he chose the smaller pile. Which pile did he choose, and how many cubic feet of wood did he lose by his ignorance? How many cord feet? If the wood was worth $4 per cord, how many dollars' worth did he lose?

37. How many more cubic feet will a bin 18 ft. long, 8 ft. wide, and 6 ft. high contain, than one 21 ft. long, 9 ft. wide, and 3 ft. high?

38. If a block of wood 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. thick is worth 72 cents, how many cents is a block 9 ft. long, $2\frac{1}{4}$ ft. wide, and $1\frac{2}{3}$ ft. thick worth?

39. Mr. Bowen has a pile of wood 9 ft. long, 4 ft. wide, and 5 ft. high. Mr. Jackson has a pile the width and height of which are the same with Mr. Bowen's, but the length is twice as great. How many times as many cubic feet are there in Mr. Jackson's pile as in Mr. Bowen's? How many cubic feet in each?

40. Mr. Gordon has a pile of wood whose height and length are the same as the height and length of Mr. Jackson's, but whose width is twice as great. How many times as many cubic feet does it contain as Mr. Jackson's? as Mr. Bowen's? How many cubic feet does it contain?

41. Mr. Kingman has a pile of wood whose length and width are the same as the length and width of Mr. Gordon's, but whose height is twice as great. How many times as many cubic feet does it contain as Mr. Gordon's? as Mr. Jackson's? as Mr. Bowen's? How many cubic feet does it contain?

## SECTION XVIII.

As skill in reckoning the currency of his country is indispensable to the business man, and indeed to every one, we insert the following exercises on Federal Money. We give them their present place because many of them will illustrate the principles of the following section, and, therefore, furnish an excellent introduction to it.

**A.** 1. How many cents are equal to 5 dimes? to 6? to 9? to 4? to 3?

2. How many dimes are equal to 4 dollars? to 7? to 6? to 8? to 2?

3. How many mills are equal to 7 cents? to 5? to 3? to 9? to 4?

4. How many cents are equal to 1 dollar? to 7? to 9? to 8? to 6?

How many cents are equal in value —

5. To 3 dimes and 4 cents?
6. To 9 dimes and 7 cents?
7. To 1 dollar and 4 cents?
8. To 8 dollars and 3 cents?
9. To 1 dollar and 1 dime?
10. To 5 dollars and 8 dimes?
11. To 7 dollars and 4 dimes?
12. To 1 dollar 1 dime and 1 cent?
13. To 3 dollars 4 dimes and 5 cents?
14. To 4 dollars 3 dimes and 5 cents?
15. To 5 dollars 4 dimes and 3 cents?
16. To 7 dollars 6 dimes and 8 cents?

NOTE. The pupil should be able to answer the above questions, and also those under the next letter, without the slightest hesitation.

**B.** 1. How many dimes equal 20 cents? 50? 70? 90? 30?

2. How many dollars equal 40 dimes? 80? 100? 60? 50?

3. How many cents equal 50 mills? 60? 80? 20?

4. How many dollars equal 100 cents? 500? 900? 700?

5. How many dimes and cents equal 74 cents? 83? 38? 19? 91? 25? 52? 37? 43? 87? 94? 49? 54? 69? 78?

6. How many dollars and dimes are equal to 49 dimes? to 94? to 73? to 61? to 16? to 57? to 88? to 21? to 12? to 31? to 58? to 43? to 34?

7. How many cents and mills are equal to 54 mills? to 71? to 17? to 58? to 93? to 39? to 28? to 82? to 46? to 69?

8. How many dollars and cents are equal to 807 cents? to 708? to 403? to 502? to 205? to 603? to 501? to 105?

9. How many dimes and mills are equal to 307 mills? to 603? to 508? to 603? to 105? to 804?

10. How many dollars, dimes, and cents are equal in value to 437 cents? to 734? to 347? to 581? to 671? to 251? to 152? to 843? to 348? to 627? to 999? to 329? to 888? to 469?

11. How will each of the values written under number 10 be expressed in terms of dollars and cents?

12. How many dimes, cents, and mills are equal to 341 mills? to 821? to 437? to 734? to 683? to 386? to 863? to 497? to 111? to 297? to 999?

13. How will each of the values written under number 12 be expressed in terms of dimes and mills?

**C.** By the ordinary method of writing numbers expressing values in United States money, the figures representing the dollars are placed at the left of the decimal point, according to the laws of writing numbers before explained; and those representing dimes, cents and mills are placed at the right, — the dimes in the first place, the cents in the second, and the mills in the third. At the left of the whole, the character $ is placed to show that the numbers represent values in United States money.

ILLUSTRATIONS.

1. $4. = $4.00 = *4 dollars = 40 dimes = 400 cents.

2. $.4 = $.40 = 4 dimes = *40 cents.

3. $.04 = $.040 = *4 cents = 40 mills.

4. $.44 = 4 dimes and 4 cents = *44 cents.

5. $4.44 = 4 dollars, 4 dimes, and 4 cents = *4 dollars and 44 cents = 44 dimes and 4 cents = 444 cents.

6. $4.4 = $4.40 = 4 dollars and 4 dimes = *4 dollars and 40 cents = 440 cents = 44 dimes.

7. $2.47 = 2 dollars, 4 dimes, and 7 cents = *2 dollars and 47 cents = 247 cents = 24 dimes and 7 cents.

NOTE. The following will furnish further illustrations of the effect of changing the place of the figures with reference to the point. We have expressed the value of each number as it would usually be read. Let the pupil show other forms in which the same values can be expressed.

8. $2.57 = 2 dollars and 57 cents.
9. $.257 = 25 cents and 7 mills.
10. $25.7 = 25 dollars and 70 cents.
11. $257. = 257 dollars.
12. $1.07 = 1 dollar and 7 cents.
13. $10.7 = $10.70 = 10 dollar and 70 cents.
14. $107. = $107.00 = 107 dollars.
15. $63. = $63.00 = 63 dollars.
16. $630. = $630.00 = 630 dollars.
17. $6.3 = $6.30 = 6 dollars and 30 cents.
18. $.63 = 63 cents.
19. $.063 = 6 cents and 3 mills.
20. $5. = $5.00 = 5 dollars.
21. $50. = $50.00 = 50 dollars.
22. $500. = $500.00 = 500 dollars.
23. $.5 = $.50 = 50 cents.
24. $.05 = 5 cents.
25. $.005 = 5 mills.

From the above it appears that every change that is made in the place of the figures in reference to the decimal point,—

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* The form marked with a star represents the usual method of reading these values.

whether it is made by changing the position of the point, or by writing other figures between the given figures and the point,— alters the value they represent. A figure can only alter the value expressed by other figures, when it is written between them and the point.

**D.** Express in figures the following values, viz.: —

1. 6 dollars and 18 cents. 18 dollars and 17 cents.
2. 9 dollars and 1 cent. 1 dollar and 9 cents.
3. 5 dollars and 6 cents. 14 dollars and 3 cents.
4. 83 dollars and 23 cents. 23 dollars and 83 cents.
5. 162 dollars and 23 cents. 6 dollars and 5 cents.
6. 1 dollar, 2 cents, and 1 mill.
7. 4 cents and 7 mills.
8. 3 dollars, 67 cents, and 2 mills.
9. 67 dollars, 3 cents, and 2 mills.

**E.** How will you read the following expressions?

| | | | |
|---|---|---|---|
| 1. | $16. | $5.04 | $42.60 |
| 2. | $16.00 | $50.40 | $37.17 |
| 3. | $4.78 | $504. | $49.05 |
| 4. | $47.80 | $504.00 | $42.07 |
| 5. | $478.00 | $.07 | $2.47 |
| 6. | $478. | $.006 | $312.632 |
| 7. | $23.79 | $2.007 | $217.478 |

**F.** 1. A person bought some corn for $2.33, some oats for $1.13, some rye for $3.25, some wheat for $4.17. What was the amount of his purchase? If he pays $6. in money, and the rest in wood, how many dollars' worth of wood will he have to pay?

2. Bought a gold pencil for $5.27, a watch-chain for $11.25, and a key for $2.13. What was the amount of my purchase? Gave in payment an ounce of California gold, valued at $17.50, and the rest in money. How much did I give in money?

3. Mr. Crafts had $13. He paid away $3.17 to one man and $2.21 to another. How much money had he left?

4. What is the value of $.37 + $.48 + $.63 + $.27?

5. What is the value of $1.08 + $3.07 + $2.09 + $4.25?

6. What is the value of \$9.14 — \$3.14 — \$2.63 — \$2.16?

7. What is the value of \$8.23 — \$2.17 — \$2.23 — \$1.22?

8. What is the value of \$12.48 — \$1.50 — \$2.25 — \$3.26?

9. What is the value of \$8.17 + \$3.81 + \$1.49 — \$2.31?

10. A laborer earned \$1.25 on Monday, \$1.07 on Tuesday, \$.98 on Wednesday, \$1.46 on Thursday, \$1.27 on Friday, and \$1.63 on Saturday. How many dollars did he earn in all? He spent \$.37 on Monday, \$.02 on Tuesday, \$1.49 on Wednesday, \$.28 on Thursday, \$.08 on Friday, and \$.16 on Saturday. How much money did he spend during the week? How many more dollars did he earn than he spent?

11. A man went from N. York to Boston and thence to Albany. He paid \$5 for his passage to Boston, and \$.50 for his supper on board the boat. He staid in Boston 1 day, paying \$2 for his board, and \$1.27 for incidental expenses. He then bought a rail-road ticket for Albany, for which he paid \$5. On his way to Albany he stopped over night at Springfield, paying for his supper, breakfast, and lodgings, \$1.37. What was the amount of his expenses while on the journey?

12. Mr. Dyer had \$1.37 on Monday morning. He collected a debt of \$.96, sold a bushel of potatoes for \$.87, and a barrel of apples for \$1.46, paid \$.87 for a scythe, \$.27 for a rake, received \$1.97 for the use of his horse, and paid for 9½ lb. of sugar at \$.08 per lb. How much money did he then have?

13. A farmer sold a trader 9 dozen of eggs at 15 cents per dozen, 7 lb. of butter at 19 cents per lb., and 14½ lb. of cheese at 8 cents per lb., agreeing to take his pay from the store. He took 3 gallons of molasses at \$.29 per gallon, 7 lb. of brown sugar at \$.08 per lb., and 2 lb. of crushed sugar at \$.10 per lb. 1½ lb. of Souchong tea at \$.56 per lb., ¾ lb. Young Hyson tea at \$.72 per lb., ¼ lb. of ginger at \$.28 per lb., ⅛ lb. of cloves at \$.48 per lb., ⅜ lb. of cinnamon at \$.32 per lb., ⅛ lb. of nutmegs at \$2.00 per lb. and raisins at 8 cents per lb. to make up the rest. How many pounds of raisins did he buy?

**G.** 1. What are the coins of the United States? (See Section 7, letter B.)

What is the value in cents of each of the following expressions, or, when more than a dollar, what is its value in dollars and cents?

2. 1 quarter-dollar, 2 dimes, 3 half-dimes, and 4 three-cent pieces?

3. 1 half-dollar, 5 half-dimes, and 7 three-cent pieces?

4. 2 dimes, 9 three-cent pieces, and 7 cents?

5. 1 half-dollar, 1 quarter-dollar, 3 dimes, 3 half-dimes, 3 three-cent pieces, and 3 cents?

6. 1 half-eagle, 4 dollars, 7 quarter-dollars, 8 three-cent pieces?

7. 1 gold dollar, 1 silver dollar, 1 half-dollar, 1 quarter-dollar, 1 dime, 1 half-dime, 1 three-cent piece, and 1 cent?

8. 3 quarter-eagles, 5 dollars, 5 half-dollars, 9 dimes, 7 half-dimes, and 4 three-cent pieces?

9. 7 quarter-eagles, 5 quarter-dollars, and 5 half-dimes?

10. 3 half-eagles, 3 quarter-eagles, 9 half-dollars, 9 quarter-dollars, 7 half-dimes, and 7 three-cent pieces?

11. 2 double eagles, 3 eagles, 5 quarter-eagles, 4 dollars, 4 half-dollars, 9 quarter-dollars, 7 dimes, 9 half-dimes, 8 three-cent pieces, and 6 cents?

## II.

**Note.** Many of the following examples admit of more than one answer. For instance, a debt of 6 cents can be paid with two coins, by using a half-dime and a cent, or by using 2 three-cent pieces. The pupil should find all the answers each question admits. The kind of work here required is very valuable to ensure quickness and accuracy, and to fit for the business transactions of real life.

1. If you should have coins of each kind coined in the United States, and should wish to use the smallest number of coins in the payment, how (*i. e.*, with what coins) would you pay a debt of 6 cents?

The same suppositions continuing,

2. How would you pay 25 cents? 15? 13? 93? 76?
3. How would you pay 26 cents? 16? 10? 50? 75?
4. How would you pay 99 cents? 48? 36? 79? 84?
5. How would you pay 3 cents? 8? 19? 91? 73?
6. How would you pay 67 cents? 14? 43? 37? 23?
7. How would you pay $1.25? $2.50? $7.50? $1.36?
8. How would you pay $2.63? $3.50? $4.25? $15?
9. How would you pay $3.00? $7.60? $9.81? $6.37?
10. How would you pay $2.33? $1.42? $3.96? $4.17?
11. How would you pay $1.23? $5.75? $25? $22.75?
12. How would you pay $12.60? $4.07? $1.08? $3.16?

NOTE. Bank bills of various denominations above a dollar are often used as a substitute for gold and silver coins. These bills most commonly represent some one of the following values, viz.: — 1 dollar, 2 dollars, 3 dollars, 5 dollars, 10 dollars, 20 dollars, 50 dollars, and 100 dollars; but they sometimes represent 500 dollars, and 1000 dollars.

13. If you had coins of each kind coined in the United States, and bank bills of each denomination mentioned above, and should wish to pay as much as possible of each debt in bills, and at the same time should wish to use as small a *number* of bills and of coins as possible, how would you pay a debt of $3.75?

The same suppositions continuing,

14. How would you pay $4.25? $8.37? $6.00?
15. How would you pay $7.16? $27.49? $60.?
16. How would you pay $16.84? $30.06? $95.29?
17. How would you pay $47.31? $83.24? $47.97?
18. How would you pay $76.83? $13.08? $99.86?
19. How would you pay $47.38? $27.04? $2.96?
20. How would you pay $35.31? $73.21? $99.99?
21. What would have been the answer to each of the above questions under this letter if the only *bills* used had been three, five, ten, and twenty dollar bills?

**I.** It often happens that a person wishes to pay a debt when he does not have such coins or bank bills as will enable him to pay the exact sum he owes. In such cases he usually gives coins or bank bills to make a value greater than the debt he owes, receiving in return a sum equal in value to the excess of what he has paid over what he owed. This is called making change.

1. If each of two persons have several coins of each kind coined in the United States, and wish to use in each transaction as few coins as possible, how can a debt of 19 cents be paid by one to the other?

*Ans.* By giving a quarter-dollar, and receiving in return either 2 three-cent pieces, or a half-dime and a cent; or, by giving 2 dimes, receiving in return 1 cent.

Subject to the same conditions as in the last example:

2. How can a debt of 38 cents be paid? of 63? of 49?
3. How can a debt of 22 cents be paid? of 8? of 18?
4. How can a debt of 24 cents be paid? of 78? of 99?

5. How can a debt of 90 cents be paid? of 40? of 70?
6. How can a debt of 64 cents be paid? of 44? of 43?
7. How can a debt of 86 cents be paid? of 84? of 74?
8. How can a debt of $1.14 be paid? of $2.25? of $3.50?
9. How can a debt of $7.50 be paid? of $1.47? of $1.86?
10. How can a debt of $3.14 be paid? of $13.42? of $1.67?

11. Miss Ireson bought at a store 3 yards of ribbon at 8 cents per yard, 4 papers of pins at 9 cents per paper, and a yard of muslin for 23 cents. She gave in payment a gold dollar. How much change ought she to receive, and in what coins can it best be paid?

12. Mr. Jones has 3 half-dollars, 3 half-dimes, and 4 cents. Mr. French has 2 dollars, 1 half-dollar, 1 quarter-dollar, 1 half-dime, and 4 three-cent pieces. What coins must be exchanged that Mr. Jones may pay Mr. French a debt of 15 cents? That Mr. French may pay Mr. Jones 15 cents?

13. What coins must be exchanged that Mr. Jones may pay Mr. French $.20? $.11? $1.16? $.31? $.23? $1.37? $.85?

14. What coins must be exchanged that Mr. French may pay Mr. Jones $.20? $.11? $1.16? $.31? $.23? $1.37? $.85?

15. Mr. Brooks has 2 quarter-eagles, 1 three-dollar bill, 4 silver dollars and 9 three-cent pieces. Mr. Upham has 1 half-eagle, 3 two-dollar bills, 1 gold dollar, 7 half-dimes and 2 cents. In what ways can change be so made that Mr. Brooks shall pay Mr. Upham $.20? $.40? $4.50? $.50? $5.25? $.31? $.47? $.89? $1.10? $5.58? $9.37? $12.23?

16. In what way can change be so made that Mr. Upham shall pay Mr. Brooks the sums mentioned in the last example?

17. Alfred has 10 three-cent pieces, and Edmund has 8 half-dimes. In what ways can change be so made that Alfred shall pay Edmund 1 cent? 2 cents? 11? 4? 25? 19? 5? 13? 16? 8? 17?

18. In what ways can change be so made that Edmund shall pay Alfred 11 cents? 3? 1? 24? 28? 2? 4? 13? 19?

19. Mr. Allen bought of Mr. Mason 2 gallons of molasses at $.30 per gallon, 5 lb. of sugar at $.08 per pound, 4 oz. of nutmegs at $.12½ per ounce, and 9 yards of calico at $.11 per yard. He gave in payment 2 bushels of potatoes at $.75 per bushel, 2 doz. eggs at 12 cents per doz., 1 melon worth $.13, and the balance in money. What was the balance? On coming to pay the balance he found that the only coins he had were 1

half-dollar, 1 quarter-dollar, 1 half-dime, and 2 three-cent pieces; while Mr. Mason had 4 half-dollars, 5 dimes and 2 cents. What coins must be exchanged that the debt may be cancelled?

20. Two persons have each a half-dollar, a quarter-dollar, a dime, a three-cent piece, and a cent. How can change be so made that one shall pay the other 8 cents? 16 cents? 17 cents? 18 cents? 23 cents? 61 cents? 87 cents? 69 cents?

## SECTION XIX.

INTRODUCTORY NOTE. The numbers employed in arithmetical problems are sometimes so large that it is difficult to remember them, and the results of our operations on them. When this is the case we use what is called Written Arithmetic; that is, we write the numbers we use and the results we obtain, instead of retaining them in our memory as in Mental Arithmetic. Aside from those connected with the writing of numbers, written arithmetic involves no principles, mental operations, or reasoning processes, that are not involved in mental arithmetic, and a person who fully understands the latter will have no difficulty with the former. In both, the work must be done in the mind. The only use of the slate is to relieve the memory, or to exhibit to the eye the numbers used and operations performed.

The subsequent sections are intimately connected with the preceding, and indeed contain much that has before been stated in another form. The importance of the principles seems to demand the repetition.

**A.** Any number whatever may be represented by the decimal point, and the ten characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

These characters are usually called *figures*, but sometimes *digits*, or *digit figures*. Their *names*, (one, two, three, four, five, six, seven, eight, nine, and zero, or, as it is sometimes called, nothing, cipher or naught,) are always the same, but the *value* expressed by any one of them depends upon its place with reference to the decimal point.

A figure written in the first place at the left of the point, represents as many units or ones as its name indicates; written in the second place, it represents as many tens as its name indicates; in the third place, as many hundreds; in the fourth, as many thousands; and so the value would change with each change of place in reference to the point.

Hence each place has its peculiar name; the first, second, third, and fourth places, being called, respectively, the units' place, the tens' place, the hundreds' place, and the thousands' place. The following will illustrate this.

| Thousands. | Hundreds. | Tens. | Units. | Point. |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | . |

In this example, each zero marks a place, and shows that there are none of the denomination of the place it occupies expressed in that place.

As another illustration, take the expression 2503. Here each figure marks a place, and denotes as many of the denomination of that place as its name implies; *i. e.*, they represent 2 thousands, 5 hundreds, 0 tens, and 3 units, which, as we have before learned, is read, two thousand five hundred and three.

The decimal point is often omitted in writing numbers, but in all such cases it is *understood* to belong at the right of the given number, thus making the right hand figure represent units.

The largest number we can express by a single figure is 9, by two figures is 99, and by three figures is 999; for if we add 1 to either of these numbers, it will require one more figure and place to represent the sum than it did to represent the original number.

**B.** By extending these principles, we can take as many places as we please, giving to the figure in each, ten times the value it would have if written one place further to the right. The names of the places as far as the twenty-fourth are given in the following example.

| Hundreds of Sextillions. | Tens of Sextillions. | Sextillions. | Hundreds of Quintillions. | Tens of Quintillions. | Quintillions. | Hundreds of Quadrillions. | Tens of Quadrillions. | Quadrillions. | Hundreds of Trillions. | Tens of Trillions. | Trillions. | Hundreds of Billions. | Tens of Billions. | Billions. | Hundreds of Millions. | Tens of Millions. | Millions. | Hundreds of Thousands. | Tens of Thousands. | Thousands. | Hundreds. | Tens. | Units. | Point. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0, | 0 | 0 | 0 | . |

By inspecting the above example it will be seen that the first three places are occupied by units, tens, and hundreds, — the second three by thousands, tens of thousands, and hundreds of thousands, — the third three by millions, tens of millions, and hundreds of millions, and so on. If the first three places were called, as they might be with perfect propriety, units, tens of units, and hundreds of units, and we should divide the number into periods of three figures each by commas, the first period would be the period of units; the second, the period of thousands; the third of millions; the fourth of billions, &c.

The right hand figure in each period expresses units or ones of the denomination of that period, while the second figure expresses tens, and the third, or left hand figure, expresses hundreds of that denomination.

Let the pupils now answer the following questions.

1. What is the name of the first period at the left of the point? of the second period? of the third? of the fourth? of the fifth? of the sixth? of the seventh? of the eighth?

2. What is the name of the period occupying the first, second and third places at the left of the point?
3. Occupying the fourth, fifth, and sixth places?
4. Occupying the seventh, eighth, and ninth?
5. Occupying the tenth, eleventh, and twelfth?
6. Occupying the thirteenth, fourteenth, and fifteenth?
7. Occupying the sixteenth, seventeenth, and eighteenth?
8. Occupying the nineteenth, twentieth, and twenty-first?
9. Occupying the twenty-second, twenty-third, and twenty-fourth?

Which place from the point is occupied by

10. The millions? the quintillions? the units?
11. The billions? the quadrillions? the trillions?
12. The thousands? the sextillions?

How many places are there between

13. The point and the thousands' period?
14. The point and the billions' period?
15. The point and the sextillions' period?
16. The point and the trillions' period?
17. The point and the millions' period?
18. The point and the quintillions' period?
19. The point and the units' period?
20. The point and the quadrillions' period?

In which period and in which place of the period would

21. The fourth figure from the point be?
22. The seventh? the tenth? the thirteenth?
23. The sixteenth? the nineteenth? the twenty-second?
24. The second? the fifth? the eighth?
25. The eleventh? the fourteenth? the seventeenth?
26. The twentieth? the twenty-third? the third?

27. The twenty-first? the eighteenth? the twelfth?

28. The fifteenth? the sixth? the twenty-fourth?

29. What would be the denomination of a figure in each place named in the above examples, from number 21 to number 28?

30. Where must a figure be placed to represent trillions? millions? thousands? ten-millions? hundred-trillions? units? hundreds? ten-thousands? hundred-billions? quadrillions? ten-billions? hundreds of sextillions?

**C.** The places are usually only marked by the figures occupying them, and hence, that we may determine the place of a figure so as to know the denomination it represents, it is necessary that every place between the figure and the point should be occupied by some one of the digits. The digit to be used in any intermediate place will depend on the number of units of the denomination of that place, which we wish to represent. If we wish to represent simply five hundred, we write the character 5 in the third place from the point, and 0 in the first and second places. The zeros show that the 5 belongs in the hundreds' place, and that there are no units or tens in the number, except those contained in the five hundred. If we wish to represent five hundred and twenty-seven, we place 5 in the third place as before, but the first is occupied by 7 and the second by 2, showing that the 5 represents 5 hundreds, and that there are two tens and seven units in the number besides.

How will you write 4 so that it shall represent

1. Four hundreds? four ten-thousands?
2. Four millions? four hundred-millions?

How will you write 7 so that it shall represent

3. Seven thousands? seven hundred-thousands?
4. Seven ten-millions? seven billions?

How will you write the figures 5 and 9 so that they shall respectively represent

5. Ten-thousands and thousands?
6. Hundred-thousands and ten-thousands?
7. Millions and hundred-thousands?

How will you write the figures 8, 3, and 4, so that they shall respectively represent

8. Hundred-millions, ten-millions, and millions?

9. Hundred-billions, ten-billions, and billions?
10. Millions, thousands, and units?
11. Ten-millions, ten-thousands, and units?
12. Trillions, hundred-billions, and ten-billions?
13. Ten-billions, hundred-thousands, and thousands?
14. Millions, thousands, and units?
15. Hundred-trillions, hundred-billions, and hundred-millions?

**D.** In reading a number represented by figures, we ordinarily commence at the left hand and read each period as though its figures stood alone, giving afterward the name of the period. For instance, the number 42,947,315,111,300,531. would be read in the same way and would express the same value as if written 42 quadrillions, 947 trillions, 315 billions, 111 millions, 300 thousand, and 531.

Let the pupil now read each of the following numbers.

1. 43,271.
2. 607,429.
3. 500,207.
4. 1,579,432.
5. 24,000,217.
6. 914,307,426.
7. 53,279,412.
8. 53,729,415.
9. 432,160,023.
10. 21,437,986,512.
11. 70,000,000.
12. 42,000,042,042.
13. 86,102.102.
14. 547,547,547,547.
15. 150,437,986,216.
16. 101,101,101,101.
17. 20,020,020,020.
18. 2,002,002,002,002.
19. 200,200,200,200,200.
20. 130,201,040,999,999.

**E.** 1. Represent by figures five hundred and twenty-seven thousand, four hundred and eighty-nine.

2. Eight thousand four hundred and seven.

3. Eighty-five thousand.

4. Eighty-five thousand and one.

5. Eighty-five thousand and thirty-one.

6. Nine million, eight hundred and fifty-six thousand, seven hundred and twenty-one.

7. Twelve million, twelve thousand, and twelve.

8. Four billion, eight hundred seventy-six million, five hundred and four thousand, three hundred and one.

9. Four billion, eight hundred and four million, eight hundred and four thousand, eight hundred and four.

10. Thirty-seven million, eight hundred and fifty-nine thousand.

11. Thirty-seven billion, eight hundred and fifty-nine million.

12. Thirty-seven billion, eight hundred and fifty-nine thousand.

13. Thirty-seven million, eight hundred and fifty-nine.

14. Forty billion, three hundred and forty million, four hundred and eighty-seven thousand, five hundred and nine.

15. Five billion, eight hundred and seventy-six thousand, seven hundred and forty-six.

16. Seventy-five trillion, eight hundred and seventy-six billion, four hundred and eighty-two million, four hundred and seventy-six thousand, three hundred and twenty-seven.

17. Four trillion, seven hundred and sixty-four billion, eight hundred and twenty-one million, six hundred and seventeen thousand, four hundred and fifty-one.

18. Seven hundred and twenty-five trillion, eight hundred and seventy-six billion, four hundred and three million, eight hundred and fifty thousand, and four hundred.

19. Three hundred and six trillion, eighteen billion, four hundred million, three thousand, four hundred and seventy-five.

20. Three trillion, three hundred and ninety-nine billion, three hundred and ninety-nine million, three hundred thousand, four hundred and three.

21. Eighty-seven trillion, five hundred and four billion, three hundred million, seven thousand, six hundred and seventy-five.

**F.** Combinations of numbers, wherever placed, can be read as though they stood alone, if the name of the place of the right-hand figure be given after reading the figures. For instance:—347 always stands for, and may be read as, three hundred and forty-seven of the denomination of the place occupied by the 7. To illustrate this still further, we have written opposite each of the following numbers the value expressed by 347 in that number.

1. 347,241. - - - 347 thousands.
2. 347,100. - - - 347 thousands.
3. 4,134,721. - - 347 hundreds.
4. 23,476,258,675. - 347 ten-millions.

Let the pupil give the value expressed by the 409 in each of the following numbers, and also the value expressed by the 27:

5. 40,927.
6. 568,340,927.
7. 274,090.
8. 40,927,534.
9. 8,172,764,096,134.
10. 27,409.

**G.** What is the greatest number of tens that can be taken—

1. From 1768? from 7,128,904?
2. From 306,702,537? from 20,000.?
3. From 8,576,423,721? from 100,637,052,980,112?
4. What is the greatest number of hundreds that can be taken from each of the above? of thousands? of billions*? of tens of millions? of hundreds of thousands? of tens of billions?
5. How will you express the value of each of the above in terms of tens and units?

*Ans.* 1768 = 176 tens + 8 units. 7,128,904 = &c.

How will you express the value of each of the above in terms of—

6. Hundreds and units? billions and units?
7. Tens of billions and units? thousands and units?
8. Hundreds of millions and units? millions and units?

**H.** Since, as we have seen, the figure in any place expresses ten times the value it would express if written one place further towards the right, it follows of necessity that it expresses only one-tenth of the value it would express if written one place further towards the left.

To give more full illustrations of this than we have yet done, we will compare the numbers 42 and 420. In the first number the figure 2 represents 2 units, while in the second it represents 2 tens, or ten times 2 units. In the first number the 4 represents 4 tens, while in the second it represents 4 hundreds, or ten times 4 tens. Therefore, each figure found in both numbers represents ten times the value in the second that it represents in the first, or one-tenth the value in the first that it represents in the second, and as there is no figure except zero in one that is not in the other, the value of the second number must be 10 times that of the first, and the value of the first one-tenth that of the second.

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* If any number is less than a billion, as 1768, the answer may be, 1768 is less than a billion, and, therefore, no billions can be taken from it.

Let the pupil compare in a similar manner —

1. 42 and 4200.
2. 42,000 and 42.
3. 420 and 4200.
4. 347 and 3470.
5. 520 and 5200.
6. 86,430 and 86,430,000.

Compare the values expressed by the —

7. 42 in 42 and 427.
8. 39 in 39 and 4398.
9. 28 in 128 and 432,876.
10. 497 in 1497 and 638,497.
11. 574 in 15,748 and 35,742,631.

Compare the values expressed by each of the 4's —

12. In 44; in 440; in 444.
13. In 4400; in 404; in 4444.

Apply the same principles in comparing each of the 4's —

14. In 4.4; in 4.04; in 40.4; in .404.
15. In 44.4; in 4.44; in .444.
16. In 400.4; in 40.04; in 4.004; in .4004.

**I.** It thus appears that 4, written in the first place at the *right* of the point, represents one-tenth of 4 units, which equals 4 tenths, while written in the second it represents $\frac{1}{100}$ of 4 units, or $\frac{4}{100}$; in the third, it represents $\frac{1}{1000}$ of 4 units, or $\frac{4}{1000}$; &c., with the other places.

As a similar thing would be true of any other digit, we name the places at the right of the point in the manner indicated below.

| Point. | Tenths. | Hundredths. | Thousandths. | Ten-Thousandths. | Hundred-Thousandths. | Millionths. | Ten-Millionths. | Hundred-Millionths. | Billionths. |
|---|---|---|---|---|---|---|---|---|---|
| . | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

1. Which place from the point is occupied by the millionths' figure? *Ans.* The sixth place at the right.

Which place from the point is occupied —

2. By the tenths' figure? by the thousandths'?
3. By the hundredths'? by the billionths'?
4. By the ten-thousandths'? by the hundred-thousandths'?
5. By the millionths'? by the hundred-millionths'?
6. What would be the denomination of a figure in the second

place at the right of the point? in the fifth? in the first? in the sixth? in the ninth? in the fourth? in the eighth? in the seventh?

**J.** Numbers expressed by figures written at the right of the point are read on exactly the same principles as those expressed by figures at the left. (See letters D and F.) Thus, if we wish to read .37, we observe that the right-hand figure is in the hundredths' place. We read it, then, as though written $\frac{37}{100}$. The following will furnish further illustrations of the principle involved: —

1. $.86 = \frac{86}{100}$.
2. $.086 = \frac{86}{1000}$.
3. $.0086 = \frac{86}{10000}$.
4. $.349 = \frac{349}{1000}$.
5. $.0349 = \frac{349}{10000}$.
6. $.000349 = \frac{349}{1000000}$.
7. $3.49 = \frac{349}{100} = 3\frac{49}{100}$.
8. $34.9 = \frac{349}{10} = 34\frac{9}{10}$.
9. $6.07 = \frac{607}{100} = 6\frac{7}{100}$.
10. $1.01 = \frac{101}{100} = 1\frac{1}{100}$.
11. $10.1 = \frac{101}{10} = 10\frac{1}{10}$.
12. $300.3 = \frac{3003}{10} = 300\frac{3}{10}$.
13. $30.03 = \frac{3003}{100} = 30\frac{3}{100}$.
14. $3.003 = \frac{3003}{1000} = 3\frac{3}{1000}$.
15. $.3003 = \frac{3003}{10000}$.
16. $7.008706 = \frac{7008706}{1000000} = 7\frac{8706}{1000000}$.

Let the pupil now read the following, expressing the value, in all cases where it is greater than unity, both as an improper fraction and a mixed number: —

17. .73
18. 50.05
19. .08
20. 0.0764
21. .798
22. 37.9427
23. 6.4
24. 876.5874
25. 7.37
26. 3299.4856328
27. 4.28
28. 1000000.0000001
29. .94
30. 87.203794
31. 3.06
32. 87.000000079
33. 40.7
34. .000000007
35. 21.3
36. 2006.000002006

**K.** Any fraction whose denominator is a power of ten may be expressed by writing the numerator so that its right-hand figure shall occupy the place of the same name with its denominator, and omitting the denominator. Fractions thus written are called Decimal Fractions to distinguish them from Vulgar

Fractions, or those whose numerator and denominator are both written.

Write the following in the form of decimal fractions: —

1. $\frac{7}{10}$. $\frac{8}{10}$. $\frac{27}{100}$. $\frac{46}{100}$. $\frac{8}{100}$. $\frac{2}{100}$.
2. $\frac{567}{1000}$. $\frac{32}{1000}$. $\frac{5}{1000}$. $\frac{837}{100000}$. $\frac{2}{100000}$.
3. $\frac{6}{1000000}$. $\frac{37}{10}$. $3\frac{7}{10}$. $\frac{48}{10}$. $4\frac{8}{10}$.
4. $\frac{876}{100}$. $8\frac{76}{100}$. $9\frac{5}{10}$. $\frac{806}{100}$. $8\frac{6}{100}$.
5. $80\frac{6}{100}$. $4\frac{673}{1000}$. $23\frac{7}{10}$. $2\frac{37}{100}$. $\frac{237}{100}$.
6. $4768\frac{57934}{1000000}$. $578\frac{67}{100000000}$. $300\frac{3}{1000}$.
7. $\frac{4000006}{1000000}$. $4\frac{6}{1000000}$.
8. $1000000\frac{1}{1000000}$.
9. $7846732007\frac{7846732007}{1000000000000}$.

**L.** The preceding illustrations and explanations show that removing the *figures representing a number* one place further towards the left, or, which is the same thing, removing the *point* one place to the right, *multiplies* the number represented by 10, while removing the *figures* one place to the *right*, or the *point* one place to the *left*, *divides* it by 10. A change of two places would in like manner multiply or divide by 100, of three places by 1000, &c.

These principles generalized would be stated thus: —

*To express the product of any number multiplied by any power of* 10, *remove the decimal point as many places to the right as there are zeros used in writing the given multiplier.*

*To express the quotient of a number divided by any power of* 10, *remove the decimal point as many places to the left as there are zeros used in writing the given divisor.*

*When, by such change, any places between the number and point are left vacant, they must be filled by zeros.*

How will you express in figures the results of the following indicated operations?

1. 87 × 10.* 143 × 10. 8.7 × 10.
2. .87 × 10. 1.43 × 10. .087 × 10.
3. .0087 × 10. .0143 × 10. 87 ÷ 10.
4. 14.3 ÷ 10. 8.7 ÷ 10. .87 ÷ 10.
5. .087 ÷ 10. .143 ÷ 10. .0087 ÷ 10.

---

* The pupil should remember that when the decimal point is not marked, it is always understood to belong at the right of the given figures.

6. $870 \div 10$. $1430 \div 10$. $57 \div 100$.
7. $5.7 \div 100$. $327.8 \div 100$. $.57 \div 100$.
8. $.057 \div 100$. $3.278 \div 100$. $.3278 \div 100$.
9. $.3278 \times 100$. $4.267 \times 1000$. $37.9 \times 100$
10. $4.5786 \times 1000$. $643.7 \div 1000$. $7 \div 100$.
11. $.4 \div 10$. $.08 \div 100$. $63 \div 1{,}000{,}000$.
12. $479.643 \times 1000$. $479.643 \div 1000$. $.607 \times 100.000$.

13. Let the pupil now tell by inspection, without changing the place of the point or re-writing the numbers, the result of the above indicated operations. Thus, 87 multiplied by 10 equals 870, &c.

He should be able to do this without the slightest hesitation.

14. Where will you place the point in order that the figures 573 may express 573 tenths? hundredths? units? tens? hundreds? thousands? thousandths? millions? millionths?

15. Where will you place the point in order that the figures 87064 may express 87064 hundreds? hundredths? ten-thousands? ten-thousandths? hundred-millions? hundred-millionths? tens? tenths?

16. Where will you place the point in order that the figures 497837 may express 497837 units? millions? billionths? hundredths? millionths? hundreds? hundred-thousandths?

**M.** 1. Reduce $\frac{7}{8}$ to a decimal fraction.

*Solution.* $\frac{7}{8} = \frac{1}{8}$ of $7 = \frac{1}{8}$ of 70 tenths $= .8$ with a remainder of $.6$; but $.6 = .60$, and $\frac{1}{8}$ of $.60 = .07$ with a remainder of $.04$; but $.04 = .040$, and $\frac{1}{8}$ of $.040 = .005$. Therefore, $\frac{7}{8} = .8 + .07 + .005 = .875$

2. Reduce $\frac{3}{8}$ to a decimal; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{4}{6}$; $\frac{3}{12}$; $\frac{1}{8}$.

NOTE. In the following examples, the pupil need not reduce to a lower denomination than millionths, even if the division does not terminate there.

3. Reduce $\frac{2}{3}$ to a decimal fraction; $\frac{5}{6}$; $\frac{3}{7}$; $\frac{7}{11}$; $\frac{6}{7}$; $\frac{7}{13}$; $\frac{5}{12}$.

4. What is the value of $457\frac{4}{7}$ expressed decimally?

*Ans.* $457.571428\frac{4}{7}$. But $\frac{4}{7}$ of .000001 is so small that it may be omitted. We should then have for an answer 457.571428.

5. What is the value of $831\frac{7}{8}$ expressed decimally? of $5\frac{7}{8}$? of $381\frac{7}{11}$? of $271\frac{4}{7}$? of $327\frac{1}{3}$? of $141\frac{3}{8}$? of $287\frac{4}{13}$?

# SECTION XX.

**A.** Addition is the process by which, having several numbers given, we find a number equal in value to all of them.

The number thus obtained is called the sum or amount. Thus, 7 + 3 + 5 would be a problem in addition, and the answer, 15, would be the sum of 7, 3, and 5.

In order that numbers may be added, it is necessary that the things they represent shall be of the same name or denomination. 2 locks and 3 keys would be neither 5 locks nor 5 keys, but since locks and keys are both things, we can change the denomination of both by calling them things; when we shall have, 2 things and 3 things are 5 things.

In like manner 2 tens and 3 units would be neither 5 tens nor 5 units; but by changing the tens to units, calling them 20 units, we shall have 2 tens + 3 units = 20 units + 3 units = 23 units. 2 shillings + 3 pence are neither 5 shillings nor 5 pence, but since 2 shillings = 24 pence, 2 shillings + 3 pence must equal 24 pence + 3 pence, or 27 pence.

For such reasons it will be found convenient, in writing large numbers for addition, to write those of the same denomination near each other, which can best be done by writing them in vertical columns, so that units shall come under units, tens under tens, &c., and pounds under pounds, shillings under shillings, pence under pence, &c. In adding, we can begin with any denomination we choose, but it will usually be more convenient to begin with the lowest, or the one at the right hand.

**B.** *Example* 1. Mr. Jones owns the following named property; a farm worth 3798 dollars, a house and garden worth 7427 dollars, three horses worth 427 dollars, one yoke of oxen worth 125 dollars, twelve cows worth 339 dollars, and 1564 dollars in money. How many dollars is he worth?

It is evident he will be worth as many dollars as there are in 3798 + 7427 + 427 + 125 + 339 + 1564.

Placing these numbers, for the sake of convenience, in the following manner,

```
$3798.
 7427.
  427.
  125.
  339.
 1564.
------
$13680.
```

and beginning with the units' column, we add thus,—4, 13, 18, 25, 32, 40 units, equal to 4 tens and 0 units. Writing 0 in the units' place in the answer, we add the 4 tens with the tens of the tens' column, thus,—4, 10, 13, 15, 17, 19, 28 tens, equal to 2 hundred and 8 tens. Writing the 8 tens, we add the 2 hundred with the hundreds of the column of hundreds, thus,—2, 7, 10, 11, 15, 19, 26 hundreds, equal to 2 thousands and 6 hundreds. Writing the 6 hundreds, we add the 2 thousands with the thousands of the column of thousands, thus,—2, 3, 10, 13 thousands, equal to 1 ten-thousand and 3 thousands, which we write.

Having added all the denominations of each number, we must have the sum of the numbers, and hence the answer, which is 13680 dollars.

*Example* 2. What is the sum of 4 cwt. 2 qr. 19 lb. 6 oz. 15 dr.; 2 cwt. 1 qr. 21 lb. 11 oz.; 3 cwt. 3 qr. 27 lb. 13 oz. 11 dr.; and 5 cwt. 2 qr. 26 lb. 14 oz. 3 dr.?

WRITTEN WORK.

| cwt. | qr. | lb. | oz. | dr. | |
|---|---|---|---|---|---|
| 4, | 2, | 19, | 6, | 15. | |
| 2, | 1, | 21, | 11, | 0. | |
| 3, | 3, | 27, | 13, | 11. | |
| 5, | 2, | 26, | 14, | 3. | |
| 16, | 3, | 11, | 13, | 13, | *Ans.* |

*Explanation.* Commencing with the drams' column, we add thus,—3, 14, 29 drams, equal to 1 oz. 13 dr. Writing the 13 drams in the answer, and adding the 1 ounce with the ounces, we have 1, 15, 28, 39, 45 ounces, equal to 2 lb. 13 oz. Writing the 13 oz., and adding the 2 lb. with the pounds, we have 2, 28, 55, 76, 95 pounds, equal to 3 qr. 11 lb. Writing the 11 lb., and adding the 3 qr. with the quarters, we have 3, 5, 8, 9, 11 quarters, equal to 2 cwt. 3 qr. Writing the 3 qr., and adding the 2 cwt. with the hundred weight, we have 7, 10, 12, 16 hundred weight, which we write. As all the denominations have now been added, we must have obtained the sum, which is 16 cwt. 3 qr. 11 lb. 13 oz. 13 dr.

**C.** We can test the correctness of the work in many ways, a few of which we will mention. One very good method is to go over the work carefully a second time in the same manner as at first. Another is to begin at the top to add instead of beginning at the bottom. This, by presenting the figures in a different order, renders it improbable that any mistake which may have been made in the first work will be repeated. Another method is to begin at the left hand to add, writing the sum of each denomination separately, as below, and then add these sums together.

*Example* 1.

| | |
|---:|---|
| $3798. | |
| 7427. | |
| 427. | |
| 125. | |
| 339. | |
| 1564. | |
| 11000. | dollars. |
| 2400. | " |
| 240. | " |
| 40. | " |
| 13680. | = Answer as before; |

so that we may suppose that the work is correct.

*Example* 2.

| cwt. | qr. | lb. | oz. | dr. | |
|---:|---:|---:|---:|---:|---|
| 4, | 2, | 19, | 6, | 15. | |
| 2, | 1, | 21, | 11, | | |
| 3, | 3, | 27, | 13, | 11. | |
| 5, | 2, | 26, | 14, | 3. | |
| 14, | 8, | 93, | 44, | 29. | |
| 14, | 0, | 0, | 0, | 0. | = sum of cwt. col. |
| 2, | 0, | 0, | 0, | 0. | = " " qr. " |
| | 3, | 9, | 0, | 0. | = " " lb. " |
| | | 2, | 12, | 0. | = " " oz. " |
| | | | 1, | 13. | = " " dr. " |
| 16, | 3, | 11, | 13, | 13. | = Answer; |

which being the same as the one before obtained, we may suppose that the work is correct.

Another method is to begin at the left, and proceed as follows. The sum of the thousands' column, in example 1, is 11 thousand; but as in the answer first obtained there are 13 thousands, two thousands must have been obtained from the sum of the lower denominations, or else the answer is wrong. But 2 thousands equal 20 hundreds, and there are 6 hundreds written in the hundreds' place; hence we should expect to find 26 hundreds in the sum of the hundreds' column. Since there are but 24 hundreds in that sum, 2 hundreds must have been obtained from the lower denominations, or else the work is incorrect. But 2 hundreds equal 20 tens, and 8 tens are written in the tens' place; hence we should expect to find 28 tens in the sum of the tens' column. Since there are but 24 tens in that sum, 4 tens must have come from the column of units; 4 tens equal 40 units, and 0 being written in the units' place, we should expect to find 40 units in the sum of the units' column, and as that sum is 40, the work may be presumed to be right.

In example 2, the sum of the cwt. column is 14 cwt., but as in the answer first obtained there are 16 cwt., 2 cwt. or 8 qr. must have been obtained from the sum of the lower denominations, or else the work is wrong. 8 qr. + 3 qr. = 11 qr., which we must account for. As the sum of the quarters' column is only 8 qr., 3 qr. or 84 lb. must have been brought from the lower denominations. 84 lb. + 11 lb. = 95 lb., which we must account for. As the sum of the pounds' column is only 93 lb., 2 lb. or 32 oz. must have been brought from the lower denominations, or else the work is wrong. 32 oz. + 13 oz. = 45 oz., which we must account for. As the sum of the ounces' column is only 44 oz., 1 oz. or 16 dr. must have been brought from the drams, or else the work is wrong. 16 dr. + 13 dr. = 29 dr., which we must account for; and as the sum of the drams is just 29 drams, we may infer that our work is correct.

If, by any of these methods, we obtain a different result from the one we first obtained, we may be sure there is an error in one or both operations, and should examine both carefully to find the error or errors.

Some method of proof should always be resorted to, until the pupil acquires sufficient skill to be sure of the accuracy of his work without.

**D.** When numbers used to express any value, express it in terms of a single denomination, as in pence, in shillings, in dollars, or in bushels, they are called simple numbers; but when they express it in terms of several different denominations they are called compound numbers.

The numbers in example 1 are simple, but those in example 2 are compound.

The principles involved in Simple Addition (the addition of simple numbers) are exactly the same as those involved in Compound Addition (the addition of compound numbers).

**IMPORTANT NOTE.** The method given in this section illustrates the similarity of simple and compound addition. As soon as understood, it should be extended so as to include the addition of two, three, or more columns at a time, (See Section 6th, Letters C. and G.) and in compound addition the reductions should be made on the principles illustrated in Section 7th, Letters R. and S.

The pupil should master all the methods, and then, in each particular case, choose the one which seems best adapted to that case. One process may be used to prove the correctness of a result obtained by some other.

Add the numbers in the following examples : —

**1.**

137.23
829.41
327.68
548.37

**2.**

8794.3
976.2
4783.
9370.5

**3.**

206.87
4396.2
45.89
643.27

**4.**

597.63
8279.
5.864
73.82
5.6

**5.**

289375.
594271.
864385.
999999.
876378.

**6.**

.076439
.62831
4.742637
9.004
.57832

**7.**

| yd. | qr. | na. |
|---|---|---|
| 27, | 2, | 3. |
| 18, | 3, | 1. |
| 17, | | 2. |
| 43, | 2, | 3. |

**8.**

| £. | s. | d. | qr. |
|---|---|---|---|
| 84, | 17, | 8, | 1. |
| 97, | 13, | 2, | 7. |
| 48, | 6, | 11, | 1. |
| 83, | 19, | 10, | 3. |

**9.**

| bu. | pk. | qt. |
|---|---|---|
| 685, | 2, | 7. |
| 394, | | 6. |
| 821, | 3, | 5. |
| 943, | 2, | 7. |

**10.**

| £. | s. | d. | qr. |
|---|---|---|---|
| 227, | 14, | 10, | 1. |
| 7, | 18, | 4, | 3. |
| 28, | 10, | 9, | 2. |
| 478, | 19. | | |
| 26, | 17, | 11, | 1. |
| 478, | 13, | 8, | 3. |

**11.**

| T. | cwt. | qr. | lb. | oz. | dr. |
|---|---|---|---|---|---|
| 3, | 13, | 2, | 19, | 13, | 12. |
| 1, | 18, | 1, | 25, | 15, | 7. |
| 9, | 14, | 1, | 27, | 5, | 8. |
| 13, | 19, | 3, | 14, | 11, | 14. |
| 4, | 8, | | 20, | 6, | 15. |
| 23, | 17, | 3, | 22, | 13, | 8. |

**12.***

| rd. | yd. | ft. | in. |
|---|---|---|---|
| 4, | 4, | 2, | 7. |
| 8, | 3, | 1, | 11. |
| 2, | 5, | 2, | 4. |
| 1, | 4, | 1, | 10. |
| 2, | 5, | 0, | 9. |
| 3, | 3, | 2, | 6. |

**13.†**

| rd. | yd. | ft. | in. |
|---|---|---|---|
| 8, | 2, | 1, | 11. |
| 3, | 5, | 2, | 6. |
| 4, | 3, | 1, | 9. |
| 7, | 2, | 0, | 8. |
| 3, | 4, | 2, | 10. |
| 7, | 2, | 2, | 3. |

**14.**

| m. | fur. | rd. | yd. | ft. | in. |
|---|---|---|---|---|---|
| 7, | 6, | 37, | 4, | 2, | 8. |
| 9, | 4, | 24, | 2, | 1, | 2. |
| 6, | 7, | 34, | 1, | 2, | 3. |
| 8, | 5, | 28, | 1, | 1, | 7. |
| 9, | 3, | 37, | 4, | 2, | 8. |
| 7, | 4, | 19, | 1, | 1, | 6. |

**15.**

| rd. | yd. | ft. | in. |
|---|---|---|---|
| 9, | 3, | 2, | 4. |
| 8, | 4, | 2, | 1. |
| 9, | 5, | 1, | 10. |
| 2, | 4, | 2, | 11. |
| 3, | 2, | 2, | 6. |

**16.**

| ℔. | ℥. | ʒ. | ℈. | gr. |
|---|---|---|---|---|
| 6, | 8, | 5, | 2, | 16. |
| 9, | 8, | 4, | 1, | 17. |
| 3, | 9, | 2, | 2, | 8. |
| 7, | 3, | 7, | 1, | 19. |
| 4, | 6, | 6, | 2, | 13. |
| 9, | 8, | 3, | 2, | 15. |

**17.**

| A. | R. | sq. rd. | sq. yd. | sq. ft. | in. |
|---|---|---|---|---|---|
| 16, | 2, | 28, | 19, | 7, | 132. |
| 14, | 3, | 37, | 8, | 2, | 47. |
| 66, | 1, | 19, | 17, | 3, | 142. |
| 18, | 2, | 39, | 16, | 4, | 27. |
| 17, | 3, | 21, | 28, | 8, | 99. |
| 25, | 2, | 31, | 30, | 6, | 100. |

| 18. | 19. | 20. |
|---|---|---|
| | .7037 Acres. | .0064 |
| $8732.175 | 2.96 | .206 |
| 243.26 | .08972 | 9.04 |
| 11.54 | 5.861 | 87.63214 |
| 52.36 | .407967 | 3.8006 |
| 8.24 | 82.2731 | 70.3 |
| 497. | 25.7 | 4.857632 |
| 57.438 | 4.8063 | 428. |

21. How many are 831 + 427 + 1058 + 69486 + 37 + 499 + 54638 ?

---

* Obtaining the answer to the 12th example in the usual method, we shall find it to be 24 rd. 5 yd. 2 ft. 11 in. This is correct, but it is not in the best form, for although there are not units enough expressed of any denomination to make one of the next higher, it equals 25 rd. 1 ft. 5 in. Show the truth of this statement, and show also *why* the answer does not *at first* appear in the best form.

† The answer to the 13th example will take the form at first of 35 rd. 4½ yd. 2 ft. 11 in., which should be changed, for the sake of simplicity, to 35 rd. 5 yd. 1 ft. 5 in. Show the equality of the two expressions, and the method by which the reduction can be made.

22. 58.7 + 3.37 + 42. + 54.6 + .32 + 87.43 + 98.27 ?

23. \$8.75 + \$9.72 + \$4.83 + \$27. + \$8.07 + \$47.43 + \$91.24 ?

24. \$87.50 + \$647. + \$8.54 + \$.73 + \$.025 + \$87.631 + \$42.968 ?

25. \$3976.43 + \$18.275 + \$867. + \$4.04 + \$.07 + \$87894.325 ?

26. 7698 + 43279 + 86372 + 48563 + 19048 + 25407 ?

27. 104376 + 298543 + 724187 + 436794 + 827632 + 549731 + 286423 + 379485 + 2176369 + 453642 + 815437 ?

28. 8067.134 + 538.0076 + 91.0789 + 4736.89 + 25.9843 + 5813.0076 + 4257.321 + 879.5207 + 6007.006 ?

29. 7386.275 + 42.78 + 64.83 + 2.719 + 84.67 + 96.81 + 37.94 + 814.9 + 46.24 + 4.376 + .821 + 474.3 + 86.27 + 47.09 ?

30. 4.76 + 3.27 + 48. + 29.3 + 2.17 + 217. + 48.3 + 625. + 2.8 + 37.9 + 64.83 + 5.27 + .691 + .478 + 27.3 + 4.81 + 97.06 ?

31. 5£, 8s. 7d. 3qr. + 12£, 8s. 9d. 1qr. + 4£, 18s. 11d. 2qr. + 7£, 14s. 6d. 1qr. + 5£, 19s. 7d. 1qr. + 13£, 4s. 2d. 2qr. ?

32. 47 gal. 3 qt. 1 pt. 2 gi. + 37 gal. 1 qt. 1 pt, 1 gi. + 85 gal. 2 qt. 2 gi. + 25 gal. 2 qt. 1 pt. 3 gi. + 54 gal. 2 qt. 1 pt. 3 gi. ?

33. 15 yd. 2 qr. 2 na. + 18 yd. 3 qr. 1 na. + 27 yd. 3 qr. 3 na. + 42 yd. 1 qr. + 87 yd. 3 na. + 3 qr. 3 na. + 26 yd. 1 qr. 1 na. ?

34. 18 ℔, 6 ℥, 5 ʒ, 2 ℈, 5 gr. + 7 ℔, 8 ℥, 7 ʒ, 1 ℈, 18 gr. + 4 ℔, 11 ℥, 4 ʒ, 2 ℈, 13 gr. + 25 ℔, 9 ℥, 1 ℈, 4 gr. + 11 ℥, 1 ℈ ?

35. 17 T. 18 cwt. 3 qr. 25 lb. + 19 T. 13 cwt. 2 qr. 14 lb. + 7 T. 9 cwt. 23 lb. + 16 cwt. 17 lb. + 8 T. 3 qr. 14 lb. ?

36. 2 m. 7 fur. 28 rd. 4 yd. 1 ft. 3 in. + 6 m. 5 fur. 19 rd. 2 yd. 2 ft. 11 in. + 25 m. 4 fur. 37 rd. 5 yd. 8 in. + 94 m. 1 fur. 24 rd. 4 yd. 2 ft. 8 in. ?

37. 19 m. 5 fur. 37 rd. 2 yd. 2 ft. 2 in. + 16 m. 4 fur. 18 rd. 5 yd. 1 ft. 7 in. + 37 m. 15 rd. 2 yd. 2 ft. 8 in. + 17 rd. 5 yd. 7 in. + 3 m. 7 fur. 18 rd. 4 yd. 2ft. 9 in. ?

38. 14 A. 3 R. 28 sq. rd. 27 sq. yd. 8 sq. ft. 12 sq. in. + 27 A. 2 R. 31 sq. rd. 17 sq. yd. 5 sq. ft. 137 sq. in. + 35 A. 1 R. 31 sq. rd. 18 sq. yd. 5 sq. ft. 116 sq. in. + 21 A. 26 sq. rd. 25 sq. yd. 5 sq. ft. 107 sq. in. + 43 A. 2 R. 14 sq. rd. 19 sq. yd. + 1 R. 15 sq. rd. 37 sq. in.?

39. 18 w. 4 da. 21 h. 37 m. 5 sec. + 37 w. 5 da. 16 h. 43 m. 57 sec. + 19 w. 3 da. 14 h. 46 m. 38 sec. + 19 w. 6 da. 23 h. 56 m. 27 sec.?

40. 40 lb. 7 oz. 5 dwt. 6 gr. + 9 lb. 8 oz. 19 dwt. 22 gr. + 2 lb. 11 oz. 19 dwt. 23 gr. + 7 lb. 8 dwt. 19 gr. + 11 oz. 6 dwt. + 3 lb. 1 oz. 15 gr. + 8 lb. 17 dwt. + 3 lb. 23 gr. + 18 dwt. 7 gr. + 9 oz. 15 gr. + 7 lb. 3 oz. 13 dwt. 15 gr.?

41. A stable-keeper bought 4 loads of hay. The first load weighed 1 T. 17 cwt. 2 qr. 15 lb., the second 18 cwt. 3 qr. 34 lb., the third 1 T. 8 cwt. 1 qr. 19 lb., and the fourth 1 T. 4 cwt. 2 qr. 21 lb. How much did they all weigh?

42. He paid $21.57 for the first load, $14.98 for the second, $22.19 for the third, $19.73 for the fourth. How much did he pay for all?

43. The first load lasted him 2 w. 5 da., the second 2 w. 3 da., the third 3 w. 1 da., and the fourth 3 w. 6 da. How long did all last him?

44. An apothecary in mixing medicine used 1 ℥, 3 ʒ, 2 ℈, 5 gr. of one kind, 3 ℥, 5 ʒ, 1 ℈, 15 gr. of another, 7 ʒ, 2 ℈, 17 gr. of another, 4 ʒ, 2 ℈, of another, and 5 ʒ, 2 ℈, 17 gr. of another. What was the weight of the mixture?

45. A trader sold 9 lb. 6 oz. of sugar to one man, 8 lb. 10 oz. to another, 5 lb. 8 oz. to another, 6 lb. 10 oz. to another, 3 lb. 4 oz. to another, 2 lb. 2 oz. to another, 1 lb. 5 oz. to another, and 8 lb. 6 oz. to another. How much did he sell in all?

46. He received $.755 for the first lot, $.605 for the second, $.40 for the third, $.485 for the fourth, $.23 for the fifth, $.15 for the sixth, $.095 for the seventh, $.585 for the eighth. How much did he receive in all?

# SECTION XXI.

**A.** All arithmetical questions in which we are required to find the difference between two numbers, or the excess of one number over another, are questions in subtraction, and the process by which we determine the answers to them is called the process of Subtraction. The following are questions in Subtraction:

1. Henry found 27 apples and gave away 9 of them. How many had he left?
2. How many are 8 — 5?
3. 7 from 12 leaves how many?

Subtraction, then, may be defined to be a process by which we find how many units there are in one given number more than in another.

It may also be defined to be a process by which we find a number equal in value to the difference between two given numbers.

The larger given number, or one from which we subtract, is called the *Minuend*, the smaller given number, or one subtracted, is called the *Subtrahend*, and the result obtained is called the *Difference* or *Remainder*. In example 1, 27 is the minuend, 9 is the subtrahend, and 18 is the difference or remainder.

The minuend and subtrahend must represent things of the same kind, otherwise the subtraction cannot be performed. 5 apples from 7 apples leave 2 apples, and 5 pears from 7 pears leave 2 pears; but it would be absurd to talk of taking 5 pears from 7 apples, or 5 apples from 7 pears. We cannot subtract 5 cents from 7 dimes, but if we should exchange one of the dimes for its value in cents, we should have 6 dimes and 10 cents, from which if we should subtract 5 cents, there would be 6 dimes and 5 cents left.

We cannot subtract units from tens, but we can find how many units a given number of tens is equal to, and then subtract from that number of units.

**B.** The following examples will illustrate some of the principles to be kept in view in subtracting large numbers: —

1. Mr. Morse has in his possession 4697 dollars, and he owes 3265 dollars. How many dollars would he have left if he should pay his debts?

*Solution.* If he has 4697 dollars, and owes 3265, he would have as many dollars left as there are units in the difference between 4697 and 3265. We can best find this difference by considering the denominations separately. 5 units from 7 units leave 2 units; 6 tens from 9 tens leave 3 tens; 2 hundreds from 6 hundreds leave 4 hundreds; 3 thousands from 4 thousands leave 1 thousand. Having thus subtracted the numbers in all the denominations, we know that the remainder must be 1 thousand, 4 hundred, 3 tens, and 2 units, or that Mr. Morse would have 1432 dollars after paying his debts.

In performing the above subtraction the work could have been commenced as well at any other denomination as at the units.

*Example* 2. A man who had 17£ 8s. 9d. paid away 3£ 4s. 6d. How much had he remaining?

The usual reasoning process shows that he would have as many £, s., and d. left as there are in the difference between 17£ 8s. 9d. and 3£ 4s. 6d.

6d. from 9d. = 3d. 4s. from 8s. = 4s. 3£ from 17£ = 14£.

The answer is, therefore, 14£, 4s. 3d.

Or, beginning at the left, 17£ — 3£ = 14. 8s. — 4s. = 4s. 9d. — 3d. = 6d. *Ans.* 14£, 4s. 3d.

**C.** From the nature of subtraction, it is evident, that if the minuend were divided into two parts, such that one should equal the subtrahend, the other would equal the remainder. We can, therefore, test the correctness of the work, by adding the subtrahend and remainder, observing whether the amount be equal to the minuend or not. If it be, the work is probably correct; but if it be not, there is an error either in the subtraction or addition, and possibly in both.

Another method of proof, depending upon the same principles as the last, is to subtract the remainder from the minuend; — the remainder thus obtained should equal the subtrahend.

Although the result is not affected by the manner of writing the numbers, it is convenient to place those of the same denomination near each other, or to write units under units, tens under tens, &c., in simple numbers, and pounds under pounds, shillings under shillings, yards under yards, feet under feet, &c., in compound.

For the sake of uniformity, we usually place the minuend above the subtrahend, and the remainder beneath; as in the following written work of examples 1 and 2: —

4697. Minuend.
3265, Subtrahend.
1432. Remainder.
4697. Sum of Subtrahend and Remainder = the Minuend.
3265. Difference of Rem. and Minuend = the Subtrahend.

| £ | s. | d. | |
|---|---|---|---|
| 17, | 8, | 9. | Minuend. |
| 3, | 4, | 6. | Subtrahend. |
| 14, | 4, | 3. | Remainder. |
| 17, | 8, | 9. | Sum of the Sub. and Rem. equal to the Minuend. |
| 3, | 4, | 6. | Difference of Rem. and Min. equal to Subtrahend. |

**D.** 1. How many are 4736 — 3214? 81948 — 30521?

2. How many are .0769 — .0316? 200.739 — 100.428?

What is the value of the following?

3. 27 bu. 3 pk. 7 qt. 1 pt. — 14 bu. 2 pk. 3 qt. 1 pt.

4. 3 lb. 11 oz. 15 dwt. 18 gr. — 1 lb. 8 oz. 13 dwt. 4 gr.

5. 6 T. 17 cwt. 2 qr. 26 lb. 13 oz. 11 dr. — 2 T. 6 cwt. 15 lb. 8 oz. 3 dr.

6. 16 ℔, 8 ℥, 5 ʒ, 2 ℈, 18 gr. — 5 ℔, 4 ℥, 2 ʒ, 1 ℈, 6 gr.

7. 1757 gal. 3 qt. 1 pt. 3 gi. — 1323 gal. 1 qt. 2 gi.

8. 18 rd. 4 yd. 2 ft. 11 in. — 6 rd. 2 yd. 1 ft. 5 in.

9. 4207.6539 yds. — 3105.231 yds.

10. 9406 yds. 3 qr. 2 na. 2 in. — 2103 yds. 1 qr. 1 na. 1 in.

11. .007639 Tons — 007214 Tons.

12. A trader bought a box of sugar, weighing 3 cwt. 1 qr. 27 lb. How much did he have left after selling 1 cwt. 1 qr. 13 lb.?

13. A farmer bought a piece of land containing 38 A. 2 R. 35 rd., and afterwards sold 7 A. 1 R. 27 rd. How much did he have left?

14. Mr. Jackson spent 43£, 18s. 9d. 3qr. for travelling expenses in England, and 27£, 6s. 8d. 1qr. for travelling expenses in Scotland. How much more did he spend in England than in Scotland?

15. He travelled 2379.86 miles in England, and 1208.31 miles in Scotland. How many more miles did he travel in England than in Scotland?

16. A silver-smith bought 1 lb. 6 oz. 7 dwt. of silver of one man, 2 lb. 3 oz. 18 dwt. 23 gr. of another, and 3 lb. 11 oz. 13 dwt. 21 gr. of another. How much did he buy of all? How much will he have left, after manufacturing and selling 3 lb. 4 oz. 12 dwt. 16 gr. of it?

**E.** It is often the case that a figure in the subtrahend represents a greater value than the corresponding figure of the minuend. In that case we take one of a higher denomination in the minuend, reduce it to the required denomination, adding its value to the value of the figure already expressed, and subtract from the sum thus obtained the value of the subtrahend figure.

Suppose, for example, that we wish to find the difference between 62.7 and 35.86.

They may be written as below.

| | |
|---|---|
| 5, 11.16,10 | Minuend, changed in form. |
| 6 2 . 7 | Minuend. |
| 3 5 . 8 6 | Subtrahend. |
| 2 6 . 8 4 | Difference. |

*Explanation of process.* As there are no hundredths expressed in the minuend, we reduce one of the 7 tenths to hundredths, leaving 6 tenths. 1 tenth = 10 hundredths, from which subtracting 6 hundredths, leaves a remainder of 4 hundredths. As 8 tenths cannot be subtracted from 6 tenths, we reduce one of the 2 units to tenths, leaving 1 unit. 1 unit = 10 tenths, which added to the 6 tenths left in the tenths' place, equal 16 tenths; 8 tenths from 16 tenths = 8 tenths. As 5 units cannot be taken from the 1 unit left in the units' place, we reduce one of the 6 tens to units, leaving 5 tens. 1 ten = 10 units, which added to 1 unit equal 11 units; 5 units from 11 units = 6 units. 3 tens from 5 tens leave 2 tens.

The answer, then, is 2 tens, 6 units, 8 tenths and 4 hundredths, or 26.84.

This may be proved in the same way that the preceding examples were.

*Questions on the above.*

Which expresses the larger number, 7 tenths, or 6 tenths and 10 hundredths, and why? 2 units and 6 tenths, or 1 unit and 16 tenths? 5 tens and 1 unit, or 4 tens and 11 units? 6 tens, 2 units, and 7 tenths, or 5 tens, 11 units, 16 tenths, and 10 hundredths?

How then will the remainder, obtained by subtracting 35.86 from 62.7, compare with the remainder obtained by subtracting it from 5 tens, 11 units, 16 tenths, and 10 hundredths?

*Example* 2. Find the difference between 14 lb. 6 oz. 5 dwt. 7 gr. and 8 lb. 7 oz. 18 dwt. 23 gr.

| | | | | |
|---|---|---|---|---|
| 13, | 17, | 24, | 31. | Minuend changed in form. |
| lb. | oz. | dwt. | gr. | |
| 14, | 6, | 5, | 7. | Minuend. |
| 8, | 7, | 18, | 23. | Subtrahend. |
| 5, | 10, | 6, | 8. | Remainder. |

*Explanation.* As we cannot subtract 23 gr. from 7 gr., we take 1 dwt. from the 5 dwt., which reduced to grains and added to the 7 gr. equals 31 gr.; 23 gr. from 31 gr. = 8 gr. But as 18 dwt. cannot be taken from the 4 dwt. left in the minuend, we take 1 oz. from the 6 oz. and reduce it to pennyweights; 1 oz. = 20 dwt., which added to the 4 dwt. equal 24 dwt.; 18 dwt. from 24 dwt. = 6 dwt. As 7 oz. cannot be taken from the 5 oz. left in the minuend, we take 1 lb. from the 14 lb. and reduce it to ounces; 1 lb. = 12 oz., which added to the 5 oz. equal 17 oz.; 7 oz. from 17 oz. = 10 oz. 8 lb. from 13 lb. = 5 lb.

The answer is, therefore, 5 lb. 10 oz. 6 dwt. 8 gr., which may be proved as before.

*Questions.* Which expresses the greater quantity, 5 dwt. 7 gr., or 4 dwt. 31 gr., and why? 6 oz. 4 dwt., or 5 oz. 24 dwt.? 14 lb. 5 oz., or 13 lb. 17 oz.? 14 lb. 6 oz. 5 dwt. 7 gr., or 13 lb. 17 oz. 24 dwt. 31 gr.?

How would the remainder obtained by subtracting 8 lb. 7 oz. 18 dwt. 23 gr. from 14 lb. 6 oz. 5 dwt. 7 gr., compare with that obtained by subtracting it from 13 lb. 17 oz. 24 dwt. 31 gr.?

For another example take the following: How many are 83004 dollars minus 24765 dollars?

| | | | | | |
|---|---|---|---|---|---|
| 7, | 12, | 9, | 9, | 14. | =Minuend, changed in form. |
| $8 | 3 | 0 | 0 | 4. | =Minuend. |
| $2 | 4 | 7 | 6 | 5. | =Subtrahend. |

As we cannot take 5 dollars from 4 dollars, and as there are no tens or hundreds expressed in the minuend, we take 1 thousand from the 3 thousands, leaving 2 thousands; 1 thousand = 10 hundreds, and taking 1 of these hundreds to reduce to tens, we have 9 hundreds left. 1 hundred = 10 tens, and taking 1 of these tens to reduce to units we have 9 tens left. 1 ten =

10 units, which added to the 4 units in the units' place = 14 units; subtracting, we have

9 units from 14 units = 5 units.

6 tens from 9 tens = 3 tens.

7 hundreds from 9 hundreds = 2 hundreds.

4 thousands cannot be taken from 2 thousands, therefore we take 1 ten-thousand from the 8 ten-thousands in the minuend, leaving 7 ten-thousands. 1 ten-thousand = ten thousands, which added to the 2 thousands in the thousands' place = 12 thousands.

4 thousands from 12 thousands = 8 thousands.

2 ten-thousands from 7 ten-thousands = 5 ten-thousands.

The remainder is, therefore, 58239 dollars.

*Questions upon the above.* How can it be shown that 3004 is equal to 2 thousands, 9 hundreds, 9 tens, and 14 units? That 83004 is equal to 7 ten-thousands, 12 thousands, 9 hundreds, 9 tens, and 14 units?

*Example* 4. A farmer took 8 bu. 3 pk. 5 qt. of corn from a bin containing 17 bushels. How many bushels, pecks and quarts remained?

The usual reasoning process shows that 17 bu. is the minuend, and 8 bu. 3 pk. 5 qt. the subtrahend.

| | | | |
|---|---|---|---|
| 16, | 3, | 8. | Minuend, changed in form. |
| bu. | pk | qt. | |
| 17. | | | Minuend. |
| 8, | 3, | 5. | Subtrahend. |
| 8, | 0, | 3. | Remainder. |

*Explanation.* As there are no pecks or quarts expressed in the minuend, we take 1 bu. from the 16 bu. and reduce it to lower denominations. 1 bu. = 3 pk. 8 qt. Therefore, 17 bu. = 16 bu. 3 pk. 8 qt. The subtraction can now be performed as before.

5. How many are 3743 — 2554?
6. How many are 839.74 — 213.78?
7. How many are 37.9623 — 21.978
8. How many are 300.67 — 25.38?
9. How many are .7235 — .0649?
10. How many are .00632 — .00274?

11. How many are 47.5 — 1.246 ?

12. How many are 971.38 — 6.85237 ?

13. How many are 32.006 — 4.739 ?

14. How many are 471.6 — 8.279 ?

15. How many are 642.0016 — 87.0329 ?

16. What is the value of 7£, 5s. 3d. 2qr.— 1£, 8s. 9d. 1qr. ?

17. What is the value of 18£, 14s. 2d. — 9£, 18s. 7d. 2qr. ?

18. What is the value of 27 bu. 2 pk. 2 qt. — 18 bu. 3 pk. 3 qt. ?

19. What is the value of 83 yd. 2 qr. 1 na. — 47 yd. 3 qr. 2 na. ?

20. What is the value of 8 cwt. 2 qr. 15 lb. 7 oz. 3 dr. — 2 cwt. 3 qr. 24 lb. 3 oz. 9 dr. ?

21. What is the value of 37 gal. 2 qt. 1 pt. 2 gi. — 12 gal. 3 qt. 1 pt. 3 gi. ?

22. What is the value of 37 lb. 2 dwt. — 4 lb. 7 oz. 5 dwt. 13 gr. ?

23. What is the value of 24 ℔, 3 ℥ — 5 ℔, 11 ℥, [illegible] ʒ, 2 ℈, 14 gr. ?

24. What is the value of 83 yd. — 2 qr. 3 na. ?

25. What is the value of 64£ — 28£, 14s. 7d. 2qr. ?

26. What is the value of 187.3 qr. 13 lb. — 67.17 cwt. 1 qr. 18 lb. 5 oz. 13 dr. ?

27. What is the value of 14 lb. — 3 lb. 6 oz. 17 dwt. 20 gr. ?

28. What is the value of 17 gal. — 2 gal. 3 qt. 1 pt. ?

29. What is the value of $876. — $43.287 ?

**F.** It is very frequently convenient to make all the reductions and changes in the minuend before beginning to subtract, as in the following example : —

What is the difference between 8 rd. 3 yd. 1 ft. 4 in., and 2 rd. 4 yd. 2 ft. 5 in. ?

*Explanation.* Since there are more yards, feet and inches expressed in the subtrahend than in the minuend, we will take 1 rd. from the 8 rd. and reduce it to lower denominations. 1 rd. = 5½ yd. = 5 yd. 1 ft. 6 in., which added to the 3 yd. 1 ft. 4 in. equal 8 yd. 2 ft. 10 in. Therefore, 8 rd. 3 yd. 1 ft. 4 in. — 2 rd. 4 yd. 2 ft. 5 in. = 7 rd. 8 yd. 2 ft. 10 in. — 2 rd. 4 yd. 2 ft. 5 in.

| | | | |
|---|---|---|---|
| 7, | 8, | 2, | 10. Minuend, changed in form. |
| rd. | yd. | ft. | in. |
| 8, | 3, | 1, | 4. Minuend. |
| 2, | 4, | 2, | 5. Subtrahend. |

The subtraction is performed in the usual method.

In performing the following examples, make the reductions in the same manner.

1. How many are 1400.3 — 207.5?
2. How many are 16573 — 4.878?
3. How many are 5700.001 — 1847.37?
4. How many are 217.406 — 8.909?
5. How many are 18£, 13s. 4d. 1qr. — 4£, 15s. 7d. 3qr.?
6. 3 R. 14 sq. rd. 7 sq. yd. 2 sq. ft. 19 sq. in — 23 sq. rd. 11 sq. yd. 5 sq. ft. 138 sq. in.?
7. 14 ℔, 7 ℥, 5 gr. — 6 ℔, 3 ʒ, 2 ℈, 17 gr.?
8. 15 rd. 5 yd. 2 ft. 11 in. — 16 rd. 1 ft. 4 in.?
9. 27 bu. — 8 bu. 3 pk. 7 qt.?
10. 19 lb. 4 dwt. — 8 lb. 7 oz. 4 dwt. 21 gr.?
11. 15 T. 15 cwt. 2 qr. 14 lb. 12 oz. 8 dr. — 13 T. 19 cwt. 3 qr. 27 lb. 15 oz. 15 dr.?
12. 27 sq. rd. 5 sq. ft. 17 sq. in. — 26 sq. rd. 30 sq. yd. 7 sq. ft. 53 sq. in.?
13. 6 fur. 8 rd. 1 yd. 2 ft. 1 in. — 1 fur. 8 rd. 4 yd. 2 ft. 11 in.?
14. 18 yd. 1 na. — 14 yd. 2 qr. 3 na.?
15. 6 m. 1 ft. — 5 m. 7 fur. 39 rd. 5 yd. 1 ft. 2 in.?

**G.** Examples to be performed without writing out the changed minuend. The reductions are to be made on the same principle as before, but they are to be retained in the memory instead of being written.

1. Subtract 863.72 from 947.815.
2. Subtract 728. from 4000.
3. Subtract 5.9318 from 47.
4. Subtract 9432. from 17865.3.
5. Subtract .000765 from .420006.
6. Subtract 5 lb 8 oz. 14 dr. from 3 qr. 1 lb. 11 oz. 7 dr.

7. Subtract 4 ℔, 9 ℥, 6 ʒ, 2 ℈, 13 gr. from 9 ℔, 5 ʒ, 1 ℈ 5 gr.

8. Subtract 7 fur. 3 rd. 4 yd. 2 ft. 8 in. from 1 m. 27 rd. 1 yd. 1 ft. 2 in.

9. Subtract 8 bu. 3 pk. 5 qt. from 17 bu. 2 pk. 3 qt.

10. Subtract 25 gal. 2 qt. 1 pt. from 37 gal. 2 qt. 0 pt.

Note. When in compound subtraction reductions are necessary, and the changed minuend is not written, it is usually easier to subtract from the value of the reduced unit and add the remainder to the minuend figure, or to subtract as many as possible from the written minuend figure and the rest from the value of the reduced unit, than to add first and subtract afterwards, as we have hitherto done.

Although this method is illustrated and explained in Section 7, Letter T, to which the pupil is referred, we insert a single example by way of review.

What is the value of 3 qr. 17 lb. — 1 qr. 24 lb.?

*Explanation.* On principles already explained, we take 1 qr. from the 3 qr. and reduce it to lb., which gives 28 lb. 28 lb. + 17 lb. — 24 lb. = (28 lb. — 24 lb.) + 17 lb. = 4 lb. + 17 lb. = 21 lb.

Or we may subtract 17 of the 24 lb. from the 17 lb. of the minuend, and the remaining 7 lb. from 1 qr. or 28 lb., which leaves as before 21 lb.; then taking 1 qr. from 2 qr. we have 1 qr. left.

The answer is, therefore, 1 qr. 21 lb., which may be proved as before.

11. Bought 7 T. 14 cwt. 1 qr. 19 lb. of hay, from which I sold 3 T. 7 cwt. 3 qr. 26 lb. How much had I left?

12. A goldsmith bought 7 lb. 7 oz. of gold. How much will he have left after manufacturing and selling 3 lb. 5 gr. of it?

13. A trader sold 9 yd. 3 qr. 2 na. of cloth from a piece containing 27 yd. 1 qr. 1 na. How much was left in the piece?

14. A man set out on foot to travel from Boston to Springfield, the distance being 98 miles. The first day he travelled 28 m. 7 fur. 19 rd., the second 24 m. 6 fur. 28 rd., the third 29 m. 4 fur. 36 rd. How far from Springfield at the end of the first day? at the end of the second? at the end of the third?

15. Farmer Johnson raised 87 bu. 3 pk. of oats, 127 bu. 1 pk. of Indian corn, and 64 bu. 2 pk. 5 qt. of wheat. How much grain did he raise in all? How much more oats than wheat? How

much more Indian corn than wheat? than oats? If the oats weighed 2808 lb., the Indian corn 7238 lb., and the wheat 3879.375 lb., how much did the whole weigh? How much more did the Indian corn weigh than the oats? than the wheat? How much more did the wheat weigh than the oats?

16. Mr. Smith's farm contains 24 A. 3 R. 17 sq. rd. of pasturage, and 17 A. 2 R. 25 sq. rd. of tillage. How many more acres of pasturage does it contain than of tillage? It also contains 48 A. 1 R. 15 sq. rd. of woodland. How many more acres of woodland does it contain than of pasturage? than of tillage? How many more acres of woodland than of tillage and pasturage together? How many acres are there in his farm? If his pasturage is worth $1225.375, his tillage worth $1757.16, and his woodland worth $1520.875, how much is his farm worth? How much more is his woodland worth than his pasturage? How much more is his tillage worth than his woodland? than his pasturage? He sold 6 A. 2 R. 29 sq. rd. of pasturage, 8 A. 16 sq. rd. of tillage, and 17 A. 3 R. 37 sq. rd. of woodland. How many acres of each kind did he have left?

17. A man undertook to dig a ditch for a certain price per rod. On completing it he demanded payment for a ditch 37 rd. 0 ft. 3 in. long. His employer, doubting his honesty, measured it, and found it to be but 36 rd. 5 yd. 1 ft. 9 in. long. A dispute arising between them, they called in Mr. Jenks to settle it, agreeing to abide by his decision. He measured the ditch, and found its length to be 36 rd. 16 ft. 9 in. What was the difference in their measurements?

**H.** We can begin at the left to subtract as well as at the right, if we are only careful to reserve one for reduction from each denomination in the minuend when it is required by the lower denominations. This reduction will be necessary when the figures in the subtrahend at the right of the denomination considered, are greater than the corresponding ones of the minuend.

*Example.* How many are 508.935 — 249.748?

508.935 = Minuend.
249.748 = Subtrahend.
259.187 = Remainder.

*Explanation.* Reserving 1 hundred from the 5 hundreds, we have, 2 hundreds from 4 hundreds = 2 hundreds. Reducing the 1 hundred reserved, to tens, and reserving 1 ten for further reduction, we have, 4 tens from 9 tens = 5 tens. Reducing the 1 ten to units and adding 8 units, we have, 9 units from 18 units = 9 units. Reserving 1 tenth, 7 tenths from 8 tenths = 1 tenth. Reducing the 1 tenth reserved to hundredths and adding 2 hundredths, (1 hundredth being reserved,) we have, 4 hundredths from 12 hundredths = 8 hundredths. Reducing 1 hundredth to thousandths, and adding the 5 thousandths, we have, 8 thousandths from 15 thousandths = 7 thousandths. The answer is, therefore, 259.187.

Perform the following subtractions by beginning at the left.

1. 89704 — 29821. 521.732 — 23.547.
2. 85.1 = 22.563. 42.736 — 5.749.
3. $56 — $8.73. $65.28 — $47.
4. 800635 — 32070. .4103 — .00627.

Note. As soon as the process is sufficiently well understood to justify it, the explanations should be omitted, and the results only named. Thus, in the example explained above, the pupil should say, 2 hundreds, 5 tens, 9 units, 1 tenth, 8 hundredths, 7 thousandths. The answer is, therefore, 259.187.

It is also a good mental exercise to read the results at once by inspecting minuend and subtrahend. Having the numbers to be subtracted written thus, —

8274.01 = Minuend,
2357.23 = Subtrahend,

perform the subtractions mentally, beginning at the left, and read at once 5916.78. Practice will make this very easy.

Subtract by the method last explained,

5. 473.81 from 2542.73; 81.425 from 6983.24.
6. 32.792 from 48.6; 430.062 from 621.05.
7. 314.378 from 617.83; 29.83 from 471.4263.
8. 197 A. 3 R. 27 sq. rd. 15 sq. yd. 5 sq. ft. 97 sq. in. from 231 A. 19 sq. rd.
9. 113°, 47′, 25″ from 127°, 18′, 14″.
10. 213 T. 15 cwt. 23 lb. 11 oz. 6 dr. from 307 T. 8 cwt. 2 qr. 23 lb. 8 oz. 12 dr.
11. 431 yd. 2 qr. 3 na. 2 in. from 527 yd. 1 qr. 2 na. 1 in.
12. 784£, 7d. — 327£, 14s. 8d. 3qr.

A good method of proceeding when several numbers are to be subtracted, is illustrated in the following example.

How many are 862 — 28 — 59 — 38 — 56 ?

Arranging as below, we subtract the sum of each column of the subtrahend from the appropriate part of the minuend, reducing and changing denominations, as before explained.

$$\begin{array}{rl} 862. & \text{Minuend.} \\ \left.\begin{array}{r} 28. \\ 59. \\ 38. \\ 56. \end{array}\right\} & \text{Subtrahends.} \\ \hline 681. & \text{Remainder.} \end{array}$$

Adding the units of the subtrahends, we have, 6, 14, 23, 31 units, which cannot be taken from 2 units. To obtain as many as 31 units, we must take 3 tens from the 6 tens. 3 tens = 30 units, which added to the 2 units equals 32 units; 31 from 32 leaves 1. Now it makes no difference whether we take 3 tens (on account of those we reduced to units) from the 6 tens, and afterwards take the tens in the tens' column of the subtrahend, or whether we take all together. Adopting the latter method, and adding the tens' column, we have 3, 8, 11, 16, 18 tens, which cannot be taken from 6 tens, but reducing 2 hundreds to tens and adding the 6 tens, we have 26 tens, from which 18 tens being taken there will remain 8 tens. 2 hundreds from 8 hundreds = 6 hundreds. The answer is, therefore, 681.

When the above is well understood, omit in practice a part of the explanation, as follows: 6, 14, 23, 31 from 32 leaves 1 unit; 3, 8, 11, 16, 18 tens from 26 tens = 8 tens; 2 hundreds from 8 hundreds = 6 hundreds.

The following form may also be taken: 6, 14, 23, 31, and 1 are 32 units; write one in the units' place; 3, 8, 11, 16, 18, and 8 are 26 tens; write 8 in the tens' place; 2 and 6 are 8 hundreds; write 6 in the hundreds' place.

Perform the operations indicated in the following examples, by the method explained above : —

1. 87642 — 273 — 4827 — 285 — 437 — 869 — 245.
2. 98402 — 2701 — 2596 — 1874 — 987 — 1283 — 5876.
3. 276.385 — 31.278 — 13.691 — 12.586 — 57.84 — 32.798 — 7.302.

4. .5927 — .03178 — .02764 — .0083 — .1794 — .00385.

5. 83.1 — 5.72 — 2.73 — .08 — .57 — .006 — .238 — 1.7 — 2.06.

6. 287000. — 328.7 — 221.3 — 344.37 — 2.851 — 117.06 — 576.823.

7. 283.587 — 1.27 — .328 — 9.063 — 57.063 — .00876 — 70.07 — .826.

8. Subtract .87 + 4.73 + 826. + 42.71 + 9.854 + 3.27 from 9012031.

9. Subtract 8837 + 1429 + 6372 + 8406 + 9785 + 4203 from 9120301.

10. Subtract 8375.94 + 276.483 + 5427.98 + 386.421 + 279.43 + 81.679 + .4237 + 4598.7 from 846271.3.

11. Mr. Ewell has in his possession $9478.63, but he owes $143.27 to Mr. Webster, $549.71 to Mr. May, $581.375 to Mr. Kingsbury, and $378.875 to Mr. Bryant. How much will he have left after paying his debts?

12. A cloth dealer bought 1497 yards of cotton cloth, 341.75 yd. of linen cloth, 247.875 yd. of broadcloth, 871.25 yd. of cassimere, and 647.38 yd. of satinet. How many yards did he buy? He sold to one man 327.75 yd. of cotton, and 87.27 yd. of linen; to another 236.84 yd. of cotton, 96.37 yd. of broadcloth, and 247.84 yd. of cassimere; and to another 468.49 yd. of cotton, 99 yd. of linen, 104.625 yd. of broadcloth, 439.75 yd. of cassimere, and 421.76 yd. of satinet. How many yards of cloth did he have left?

13. A man travelled 8725.67 miles in the following conveyances, viz.: — 1285.89 miles in rail-road cars, 876.81 miles in a canal-boat, 587.86 miles in a stage-coach, 725.18 miles on horse-back, 647.25 miles on foot, 3147.82 miles in a steamboat, and the rest in a ship. How many miles did he travel in a ship?

14. Messrs. Howes and Baker bought 27147 bushels of Indian corn. After selling, at private sale, 1438 bushels to one man, 2627 to another, 3781 to another, and 864 to another, they sold the rest at auction. How many bushels did they sell at auction? They received $719 for the first lot, $1313.50 for the second, $1890.50 for the third, $432 for the fourth, and enough to make up $13573.50 for what they sold at auction. How much did they receive for that which they sold at auction?

## SECTION XXII.

**A.** How much will 8 apples cost at 3 cents apiece?

The above is a question in what is called Multiplication. Solving it as we have heretofore solved similar examples, we should say: If one apple costs 3 cents, 8 apples will cost 8 times 3 cents, or 24 cents. To examine the nature of the operation on the numbers, let us suppose that a person ignorant of all numerical processes, except that of counting, should be called upon to solve the same question. If he had a quantity of cents, he might lay 3 in one place, 3 more in another, 3 more in another, and so go on laying 3 in a place till he should have 8 piles of 3 cents each. Since the cents in each pile would buy 1 apple, the cents in all would buy 8 apples; he might then, by counting the cents in the 8 piles, find how much the apples would cost. If he should have no cents, he might still determine the result in a similar way, by using pebbles, sticks, marks, or anything else of a like character. After learning how to *add*, he might obtain the result by adding 8 threes together. If, after having obtained the result in some way similar to the above, he should remember it, he would ever after be able, without counting or adding, to give the answer to any question requiring the amount of 3 taken 8 times. If he should learn in a similar manner the several amounts of 10 and each number below 10, taken as many times as there are units in each successive number from 1 to 10, he would learn the common multiplication table as far as ten. If he should now learn how to apply this knowledge to the decimal system of numbers, he would be master of the process of multiplication.

Multiplication, then, may be defined as a process by which we ascertain how much any given number will amount to, if taken as many times as there are units in some other given number.

The number supposed to be taken is called the multiplicand, the number showing how many times the multiplicand is supposed to be taken is called the multiplier, and the result is called the product. The multiplier and multiplicand are called factors of the product.

**B.** In the previous work, the pupil must have observed that it has made no difference with the product of two numbers, whether he has considered one or the other as the multiplier.

Thus, 6 times 4 = 4 times 6, or 6, fours = 4 sixes.
**5 times 3 = 3 times 5, or 5, threes = 3 fives.**

The principle may be proved true for all numbers, by the following arrangement of dots.

• • • • •
• • • • •
• • • • •

Considering the dots as being arranged in horizontal rows, there are 3 rows with 5 dots in each row; considering them as being arranged in vertical rows, there are 5 rows with 3 dots in each row; and reckoning in either way we include all the dots. Now, if these rows were extended in either direction, always being kept equal to each other, it is evident that the number of rows reckoned in one direction would always be equal to the number of dots that would be in a row were the rows reckoned in the other direction, and that all the dots would be reckoned in both instances. The number that represents the multiplicand when the rows are reckoned in one direction, will represent the multiplier when they are reckoned in the other, while the product or number of dots will be unaltered. Hence, it must always be true that it makes no difference with the product which of the two factors is taken for a multiplier, provided the other be taken as the multiplicand. It will generally be most convenient to consider the larger factor as the multiplicand, though not always so.

**C.** When either factor is a large number it will be well to consider its denominations separately, and if we write the results as we obtain them, it will be well to begin with the lowest denomination.

Suppose we wish to obtain the product of 75.69 multiplied by 7. The usual method is to write the numbers in some convenient way, as the following: —

75.69 = Multiplicand.
7 = Multiplier.
529.83 = Product.

*Explanation.* 7 times 9 hundredths = 63 hundredths, or 6 tenths and 3 hundredths. Writing the 3 in the hundredths' place, we reserve the 6 tenths to add to the product of the tenths by 7. 7 times 6 tenths = 42 tenths, and 6 tenths added = 48 tenths = 4 units and 8 tenths. Writing the 8 tenths, we reserve the 4 units to add to the product of the units by 7. 7 times 5 units = 35 units, and 4 units added = 39 units = 3 tens and 9 units. Writing the 9 units, we reserve the 3 tens to add to the product of the tens by 7. 7 times 7 tens = 49 tens, and 3 tens added = 52 tens, which, being our last product, we write. The result, then, is 529.83.

For further illustration take

*Example* 2. What is the product of 3£, 9s. 7d. multiplied by 8?

WRITTEN WORK.

$$\begin{array}{rrrl} \pounds & s. & d. & \\ 3, & 9, & 7 & = \text{Multiplicand.} \\ & & 8 & = \text{Multiplier.} \\ \hline 27, & 16, & 8 & = \text{Product.} \end{array}$$

*Explanation.* Beginning with the lowest denomination, we have 8 times 7d. = 56d. = 4s. 8d. Writing the 8d. and reserving the 4s. to add to the shillings of the next product, we have 8 times 9 shillings = 72 shillings, and 4 shillings from the former product added, are 76 shillings = 3£, 16s. Writing the 16s. and reserving the 3£ to add with the pounds of the next product, we have 8 times 3£ = 24£, and 3£ added = 27£, which, being the last product, we write.

**D.** We can test the truth of our result, by writing out by itself the product of the multiplication of each denomination, beginning either at the left or right, and afterwards adding these products together. The sum should equal the former product.

Below is the written work of examples 1 and 2, as proved by beginning at the left and writing each denomination of the product separately.

*Example* 1.

$$\begin{array}{rl} 75.69 \times 7 & \\ \hline 490.\phantom{00} & = \text{Product of 70 by 7.} \\ 35.\phantom{00} & = \text{Product of 5 by 7.} \\ 4.2\phantom{0} & = \text{Product of .6 by 7.} \\ .63 & = \text{Product of .09 by 7.} \\ \hline 529.83 & = \text{Prod. of 75.69 by 7} = \text{former Prod.} \end{array}$$

*Example* 2.

$$\begin{array}{lrl} 3\pounds,\ 9s.\ 7d. \times 8. & & \\ \hline 24\pounds, & = 24\pounds & = \text{Product of } 3\pounds \text{ by 8.} \\ \quad 72s. & = 3\pounds,\ 12s. & = \text{Product of 9s. by 8.} \\ \qquad 56d. & = 4s.\ 8d. & = \text{Product of 7d. by 8.} \\ \hline 24\pounds,\ 72s.\ 56d. & = 27\pounds,\ 16s.\ 8d. & = \text{Product of } 3\pounds,\ 9s.\ 7d. \text{ by 8} = \text{former Product.} \end{array}$$

Another method of proof would be to consider the multiplicand as the multiplier, and see if we can get the same result as before. The figures being in this way presented in a different order, we shall not be liable to repeat any mistake we may have made in our first work. We can either reduce and add our partial products as we did in the first method of performing the example, or write them out in full as in the second.

$$\begin{array}{r} 75.69 \\ 7. \\ \hline 529.83 \end{array}$$

Another method of proof is, after having written out the work as at first performed, to begin at the left hand, thus; 7 times 7 tens are 49 tens; but as there are 52 tens in our product, 3 tens must have come from the product of the lower denominations. 3 tens = 30 units, and adding to this the 9 units written in the units' place, we find there ought to be 39 units in the product. 7 times 5 units are only 35 units, hence, if our work be right, 4 units must have come from the product of the lower denominations. 4 units = 40 tenths, and adding to this the 8 tenths written in the tenths' place of our product, we find that there ought to be 48 tenths in the product. 7 times 6 tenths are only 42 tenths, hence, if our product be right, 6 tenths must have gone from the product of the hundredths. 6 tenths = 60 hundredths, and adding to this the 3 hundredths written in the hundredths' place, we find that there ought to be 63 hundredths in the product, and as 7 times 9 hundredths are 63 hundredths, we may infer that our work is right.

$$\begin{array}{rrr} \pounds & s. & d. \\ 3, & 9, & 7. \\ & & 8. \\ \hline 27, & 16, & 8. \end{array}$$

8 times 3£ = 24£. But as in the written product there are 27£, 3£ must have come from the lower denominations. 3£ = 60s., and adding to this the 16s. written in the shillings of the product, we find that there ought to be 76 shillings in the product. 8 times 9s. = 72s.; therefore, if the work be right, 4 shillings must have come from the lower denominations. 4s. = 48d., and adding to this the 8d. already written, we find there ought to be 56d. in the product, and as 8 times 7d. = 56d. we infer that the work is correct.

If the pupil should find by multiplying numbers in one method a result different from that he has obtained by multiplying the same numbers in some other method, he may be sure that there is an error in one operation or the other, and he should examine his work patiently till he finds it. No person who is willing to allow an error to pass undetected can be a good arithmetician.

**E.** 1. What is the product of 84687 × 4?

2. What is the product of .0078673 × 7?

3. What is the product of 237.904 × 8?

4. What is the product of 20078. × 9?

5. What is the product of .00978 × 6?

6. What is the product of 796.783 × 7?

7. What is the product of .00978 × 6?

8. What is the product of 37842 × 8?

9. What is the product of .7948 × 8?

10. What will 7 acres of land cost at $94.839 per acre?

11. 1 pound Avoirdupois of distilled water contains 27.7015 cubic inches. How many cubic inches will 8 pounds contain?

12. 1 pound Troy of distilled water contains 22.794377 cubic inches. How many cubic inches will 8 pounds contain?

13. How many cubic inches are there in 7 cubic feet? in 8? in 9? in 6?

14. Bought the following articles, viz.: 7 yd. of cloth at $2.875 per yard, 4 yd. of silk at $1.725 per yard, 6 shawls at $7.628 apiece, and 8 pairs kid gloves at $1.1875 per pair. How much did they all cost?

15. What is the product of 22£, 18s. 8d. 2qr. by 7?

| £ | s. | d. | qr. | |
|---|---|---|---|---|
| 22, | 18, | 8, | 2 | = Multiplicand. |
| | | | 7 | = Multiplier. |
| 160, | 10, | 11, | 2 | = Product. |

*Operation.*

7 times 2qr. = 14qr. = 3d. 2qr. Write 2qr.

7 times 8d. = 56d., + 3d. = 59d. = 4s. 11d. Write 11d.

7 times 18s. = 126s., + 4s. = 130s. = 6£, 10s. Write 10s.

7 times 22£ = 154£, + 6£, = 160£. Write 160£.

The product is, therefore, 160£, 10s. 11d. 2qr.

**Note.** When any denomination to be multiplied is very near a unit of the next higher, the work may frequently be much shortened by considering it a unit of that higher denomination, and subtracting for its deficiency in value. For instance, in the example above, — since

18s. = 1£ — 2s., 7 times 18s. must equal 7£ — 14s., or 6£, 6s., to which adding 4s., (from the previous product,) we have 6£, 10s., as before.

16. What is the product of 37£, 14s. 2d. 3qr. multiplied by 9? by 6?

17. What is the product of 23 lb. 4 oz. 16 dwt. 22 gr. multiplied by 8? by 4?

18. What is the product of 13 cwt. 2 qr. 17 lb. 13 oz. 9 dr. multiplied by 6? by 5?

19. What is the product of 6 T. 18 cwt. 1 qr. 26 lb. 2 oz. 1 dr. multiplied by 7? by 9?

20. What is the product of 9 bu. 3 pk. 7 qt. multiplied by 9? by 3?

21. What is the product of 4 rd. 3 yd. 2 ft. 8 in. multiplied by 8? by 6?

22. What is the product of 9 gal. 2 qt. 1 pt. multiplied by 5? by 2?

23. *What is the product of 1 m. 4 fur. 39 rd. 5 yd. 1 ft. 3 in. multiplied by 9? by 4?

24. What is the product of 8 w. 1 d. 23 h. 59 m. 56 sec. multiplied by 7? by 5?

25. What will .03 of a box of raisins cost at $2.375 per box?

*Explanation.* Since 1 box costs $2.375, .03 of a box will cost .03 of $2.375. .01 of $2.375, found by removing the decimal point two places toward the left, is $.02375, and .03 must, therefore, be 3 times $.02375.

The following form of writing the work exhibits clearly the change in the position of the point and the reason for it, and should be carefully followed by the pupil in all similar examples, till much practice has made him perfect in them:

$2.375 = cost of 1 box.

.01 of $2.375 = $.02375 = cost of .01 of a box.

3 times $ .02375 = $.07125 = cost of .03 of a box. *Ans.*

26. What will .9 of a yard of broadcloth cost at $3.25 per yd.?

27. If a bushel of tomatoes weigh 48.37 lb., what will .3 of a bushel weigh?

---

* In performing this example much labor may be saved by observing that the multiplicand is only 3 in. less than 1 m. 5 fur. Similar things can frequently be taken advantage of.

28. If a train of cars moves at the rate of 27.29 miles per hour, how far will it move in .07 of an hour?

29. If a cubic yard of distilled water weighs 1677.5 lb., how much does .09 of a cubic yard weigh?

30. If pine weighs .8 as much as an equal bulk of water, what will a cubic foot of pine weigh, a cubic foot of water weighing 62.5 lb.?

31. What will .4 of a ton of iron cost at $47.93 per ton?

32. What will .8 of a bag of coffee cost at $13.93 per bag?

33. What will .7 of a ton of coal cost at $5.875 per ton?

34. If Mr. Smith uses 9.678 tons of coal per year, and Mr. Parkhurst uses .7 as much, how many tons does Mr. Parkhurst use?

**Note.** The term *per cent* is often used in arithmetic, and in business transactions, instead of *one-hundredths*. Thus, 6 per cent has the same meaning as 6 one-hundredths. 8 per cent = .08 = $\frac{8}{100}$.

5 per cent of 340 acres = .05 of 340 acres.
6 per cent of $86.38 = .06 of $86.38.

Every number is 100 per cent of itself.

35. A teamster carted 7486 bushels of potatoes to a rail-road depot, receiving in payment 9 per cent of them. How many bushels did he receive?

36. A man who owned 378.37 acres of land gave 8 per cent of it to his son. How many acres did he give to his son?

37. A city merchant agrees to sell for a country farmer whatever produce the latter may send to him, on condition that he shall receive for his trouble 5 per cent of the money he receives for the produce. Under this arrangement he sells produce to the value of $1768.37. What ought he to receive for his trouble?

**Note.** A merchant who makes it his business to buy and sell goods for others, is called a Commission Merchant. The money he receives for his services is called his Commission. For instance: — In the last example the merchant receives a commission of 5 per cent on the value of the produce he sells. He will deduct this commission from the amount of the sales before paying the farmer.

38. A commission merchant sold cloth for a manufacturer, to the amount of $7643.79, receiving a commission of 3 per cent. What did his commission amount to? How much money ought he to pay the manufacturer?

39. A commission merchant buys goods for me to the amount of $387.46, for which he charges a commission of 2 per cent.

What will his commission amount to? How much money must I send him to pay for the goods and commission?

40. Mr. Moore borrowed some money of Mr. Boyden, agreeing to pay him, for each year's use of it, a sum equal to 6 per cent of the money he had borrowed. He borrowed $125.63, and kept it just one year. How much ought he to pay for the use of it? How much, then, ought he to pay Mr. Boyden in all?

**Note.** Money paid, as in the above example, for the use of money is called Interest. The money used is called the Principal. Interest is usually reckoned at a certain per cent of the principal for each year that it is used. The percentage paid for each year is called the Rate Per-cent. The interest and principal added together form the amount.

41. What is the interest of $137.84 for 1 year at 6 per cent?

42. What is the interest of $487.31 for 6 months at 6 per cent per year?

*Suggestion.* Since the rate for 1 year is 6 per cent, the rate for 6 months must be $\frac{1}{2}$ of 6 per cent, which is 3 per cent. The interest for 6 months will, therefore, be 3 per cent of the principal.

43. Mr. Adams borrowed of Mr. Wales $718.63, for which he agreed to pay interest at 6 per cent. At the end of 6 months he paid the principal and interest. How much did he pay?

44. What is the interest of $47.83 for 4 months at 6 per cent?

45. What will 49.67 barrels of apples cost at $3 per barrel?

*Solution.* If one barrel costs 3 dollars, 49.67 barrels will cost 49.67 times 3 dollars, which is equivalent to 3 times 49.67 dollars.

The work would be written thus —

$$\begin{array}{r l}
\$49.67 & = \text{Multiplicand.} \\
3 & = \text{Multiplier.} \\
\hline
\$149.01 & = \text{Product.}
\end{array}$$

*Another Solution.* It is evident that if each barrel should cost a dollar, all would cost as many dollars as there are barrels bought, which in this instance would be $49.67. But if, at $1 per bbl., they cost $49.67, at $3 per bbl. they would cost 3 times $49.67.

The work may be written thus —

49.67 bbl. at $3 per bbl.

$ 49.67 = cost at $1 per bbl.

3 times $49.67 = $149.01 = cost at $3 per bbl. = *Ans.*

**NOTE.** The second of the above forms exhibits more clearly than the first the change in the denomination of the 49.67, and is particularly well adapted to exhibit the changes when there are decimal fractions in multiplier and multiplicand, as the following example may show.

46. What will 43.75 pt. of beans weigh, if 1 pt. weighs .9 of a pound?

WRITTEN WORK.

43.75 pt. weighing .9 of a lb. per pt.
43.75 lb. = weight at 1 lb. per pt.
.1 of 43.75 lb. = 4.375 lb. = weight at .1 of a lb. per pt.
9 times 4.375 lb. = 39.375 lb. = weight at .9 of a lb. per pt. = *Ans.*

**NOTE.** It will in the end be a saving of labor for the pupil to write out his work for a time as fully as the work of the last two examples is written.

47. Mr. Rice has a basket which holds .7 of a bushel, and he has peaches enough to fill it 26.43 times. How many bushels of peaches has he?

48. If a man walks .06 of a mile per minute, how many miles will he walk in 87.29 minutes?

49. What will 37.49 lb. of saleratus cost at $.08 per lb.?

50. What will 178.69 bbl. of flour cost at $7 per bbl.?

51. What will 27.96 firkins of butter cost at $9 per firkin?

52. What will be the weight of 3794 cannon balls, each ball weighing 8 lb.?

53. If a soldier eats 4 lb. of meat in a week, how many pounds will 2896 soldiers eat in the same time?

54. If a paper of pins costs $.07, how much will 4978 papers cost?

55. If 1 pint of oats weighs .6 of a lb. and yields .4 of a lb. of oat-meal, how many pounds will 67.98 pints of oats weigh? How many pounds of oat-meal will they yield?

56. If a bushel of potatoes is worth $.5, how much are 133.75 bushels worth?

57. What will .3 of a cwt. of coffee cost at $14.83 per cwt.?

58. What will 14.83 lb. of tea cost at $.3 per lb.?

59. What will 8 boxes of lemons cost at $3.187 per box?

60. What will 3.187 cwt. of sugar cost at $8 per cwt.?

61. 8734 yd. = how many feet?

62. 6743 weeks = how many days?

64. 576.37 sq. yd = how many square feet?
65. 82 bu. 3 pk. 5 qt. 1 pt. = how many pints?

In such examples as the above there is no need of writing the multipliers, as they may be known from the table of the weight or measure used. In reducing to any denomination of which there are units already expressed, it will usually be more convenient to add those units at the time we make the reduction.

Method of writing the work of the last example: —

82 bu. 3 pk. 5 qt. 1 pt.
331 pk. = 82 bu. 3 pk.
2653 qt. = 82 bu. 3 pk. 5 qt.
5307 pt. = 82 bu. 3 pk. 5 qt. 1 pt.

The following abbreviated method of writing the work may be adopted after the above is perfectly familiar: —

82 bu. 3 pk. 5 qt. 1 pt.
331 pk.
2653 qt.
5307 pt. = *Ans.*

66. 97 bu. 2 pk. 7 qt. 1 pt. = how many pints?
67. 187 bu. 1 pk. 6 qt. 0 pt. 3 gi. = how many gills?
68. 238 gal. 3 qt. 1 pt. 2 gi. = how many gills?
69. 596 gal. 2 qt. 1 pt. 3 gi. = how many gills?
70. 9 ℔, 11 ℥, 6 ʒ, 2 ℈ = how many scruples?
71. 8 ℔, 3 ℥, 7 ʒ, 1 ℈ = how many scruples?
72. 937 le. 1 m. 5 fur. = how many furlongs?
73. 286 le. 2 m. 7 fur. = how many furlongs?

74. What will 37 bu. 2 pk. 3 qt. 1 pt. of cherries cost at 4 cents per pint?

75. What will 114 bu. 3 pk. 2 qt. of wheat cost at 1 cent per gill?

76. What will 17 gal. 3 qt. 1 pt. 3 gi. of oil cost at 5 cents per gill?

77. What will 93 gal. 2 qt. 1 pt. 2 gi. of brandy cost at 8 cents per gill?

**F.** It often happens that a number used as a multiplier is the product of two or more factors. In such cases it is sometimes convenient to resort to a process similar to those explained below.

How many are 12 times 743?

*Explanation.* Since 12 equals 6 times 2, 12 times a number must be equal to 6 times 2 times the number. Applying this to the above example, the work may be written thus: —

$$\begin{array}{r l}
743 & \\
2 & \\
\hline
1486 & = 2 \times 743. \\
6 & \\
\hline
8916 & = 6 \times (2 \times 743) = 12 \times 743.
\end{array}$$

Again, since $12 = 3$ times 4, 12 times a number must be equal to 3 times 4 times the number. Applying this to the above example, the work may be written thus: —

$$\begin{array}{r l}
743 & \\
4 & \\
\hline
2972 & = 4 \times 743. \\
3 & \\
\hline
8916 & = 3 \times (4 \times 743) = 12 \times 743.
\end{array}$$

Again, since $12 = 2$ times 2 times 3, 12 times a number must be equal to 2 times 2 times 3 times the number. The work of the above, thus performed, would be written as follows: —

$$\begin{array}{r l}
743 & \\
3 & \\
\hline
2229 & = 3 \times 743. \\
2 & \\
\hline
4458 & = 2 \times (3 \times 743) = 6 \times 743. \\
2 & \\
\hline
8916 & = 2 \times (6 \times 743) = 12 \times 743.
\end{array}$$

Apply these principles to the following examples: —

1. How many are 879 × 18? 6427 × 42?
2. How many are 98.74 × 27? 4.379 × 64?
3. How many are .8764 × 36? 2976.4 × 28?
4. How many are 278. × 54? 63.189 × 56?
5. How many are 579 × 20? 18.67 × 70?
6. How many are 128£, 13s. 8d. 1qr. × 35?

7. How many are 77 T. 17 cwt. 2 qr. 19 lb. 6 oz. 11 dr. × 63?

8. How many are 27 ℔, 8 ℥, 6 ʒ, 2 ℈, 17 gr. × 45?

9. How many are 24 m. 6 fur. 27 rd. 4 yd. 1 ft. 7 in. × 42?

10. How many are 19 w. 5 d. 17 h. 38 m. 29 sec. × 36?

11. What will 12 acres of land cost at 349 dollars per acre?

*Solution.* Since 12 = 2 times 6, 12 acres will cost 2 times as much as 6 acres. The process would be; if 1 acre cost $349, 6 acres will cost 6 times $349, which is found to be $2094. If 6 acres = $2094, 2 times 6 acres, or 12 acres, will cost 2 times $2094, which is found to be $4188.

The following is a convenient form for writing the work on the slate or blackboard:—

$349 = cost of 1 acre.
$2094 = " " 6 acres.
$4188 = " " 2 times 6 acres, or 12 acres.

12. How many bushels of corn are there in 18 bins, each bin containing 198 bushels? How many in 42 bins? in 16 bins?

13. How many pounds of cotton are there in 63 bales, each bale containing 437 pounds? How many in 15 bales? in 30 bales?

14. A man bought 36 bales of cloth, each bale containing 428 yards. How many yards did they all contain? He paid $753.28 for each bale. What did he pay for all? He sold it at $797.36 per bale. How much did he receive for it all? How much did he gain on the cloth?

15. A farmer had 24 stacks of hay, each stack containing 2 T. 14 cwt. 3 qr. 19 lb. How much did they all contain? If he should sell each stack for $36.274 how much would he sell them all for?

16. A jeweller bought of a California merchant 42 bars of gold, each bar weighing 6 oz. 17 dwt. 22 gr. How much did they all weigh? He paid $136.75 per bar for it. What did it all cost him?

**G.** We can find the product of two numbers by multiplying one of them by the parts into which we choose to separate the other, and then adding the products thus obtained together. For instance; 8 times 7 = 6 times 7 + 2 times 7 = 3 times 7 + 5 times 7 = 7 times 7 + once 7 = 4 times 7 + 4 times 7 = 56.

This principle, and the one illustrated in letter F., are ordinarily employed when the multiplier contains several denominations. We usually get 83 times a number, by adding together 80 times the number and 3 times the number. We get 647 times a number by adding together 600 times the number, 40 times the number, and 7 times the number. We get 8009 times a number by adding together 8000 times the number and 9 times the number. It can make no difference which part of a number we multiply by first, provided we multiply by all its parts, yet for the sake of uniformity it may be well generally to begin with the lowest denomination.

Suppose that we are required to find the product of 869 by 75. We may find the product of 869 by 5 in the usual method. We then wish to find the product of 869 by 70. Now as 70 = 7 × 10, the product obtained by multiplying by 70 must be 7 times the product obtained by multiplying by 10. But 869 multiplied by 10 = 869 tens, = 8 thousands, 6 hundreds, and 9 tens, which, multiplied by 7, will give for a product 70 times 869. This will be found to be 60830, and adding it to the product of the multiplication by 5, we have 65175, as the product of 869 by 75.

A good method of writing the work is given below: —

869. Multiplicand.
75. Multiplier.
4345. Product by 5.
60830. " " 70.
65175. " " 75.

Since the product of the multiplication by the units is sufficient to fix the place of the figures in the subsequent products, the zero at the right of the second product need not have been written. The product would then stand: —

4345. Product by 5.
6083 . " " 70.
65175. 75.

Examples in which the multiplier consists of more than two figures are performed in a similar way.

2. What is the product of 78.069 × 285?

78.069 = Multiplicand.
285. = Multiplier.
390.345 = Product by 5.
6245.52 = Product by 80.
15613.8 = Product by 200.
22249.665 = Product by 285.

3. What is the product of 80.276 × 39? of 30678 × 427?
4. What is the product of 298.794 × 148? of 5.796 × 238?
5. What is the product of 273.986 × .27? of 45.718 × 4.32?
6. What is the product of 494.3 × .0078? of .00876 × .074?
7. What is the product of 2.3879 × 27.41? of .478 × 39.65?

8. A piano-forte dealer sold in one month 57 piano-fortes for $337.50 apiece. How much did he receive for them?

9. Bought 337 bales of silk goods at $447.86 per bale. What was the cost of the whole?

10. What will be the cost of building 43.7 miles of rail-road at $29756.84 per mile?

11. If a man works on an average 2745 hours per year, how many hours will he work in 9.76 years?

12. A country trader bought 47 lb. of tea at 38 cents per lb., 28 lb. at 59 cents per lb., 7 cwt. of sugar, at $6.833 per cwt., and 9 kegs of raisins at $4.42 per keg. What was the amount of his purchase?

**II.** When the multiplier consists of more than one denomination, much labor in writing figures may be saved by applying the principles illustrated in the following examples: —

What is the product of 8356 multiplied by 79?

It is evident that, in performing the required multiplication, we shall obtain units by multiplying 6 units by 9 units. We shall obtain tens by multiplying 5 tens by 9, and 6 units by 7 tens, and we may have some from the product of the units. We shall obtain hundreds by multiplying 3 hundreds by 9, and 5 tens by 7 tens, and we may have some from the former products. We shall obtain the other denominations in a similar manner. We may then proceed thus, writing the figures of each denomination as usual: —

WRITTEN WORK.

8356. Multiplicand.
79. Multiplier.
660124. Product.

*Explanation.* 9 times 6 units = 54 units. We write 4 units. 9 times 5 tens = 45 tens, + 5 tens (from the product of the units) = 50 tens, + 7 tens times 6, or 42 tens, = 92 tens = 9 hundreds and 2 tens. We write 2 tens. 9 times 3 hundreds = 27 hundreds, + 9 hundreds (from the previous product) = 36 hundreds, + 7 tens times 5 tens, or 35 hundreds, = 71 hundreds = 7 thousand and 1 hundred. We write 1 hundred. 9 times 8 thousands = 72 thousands, + 7 thousands (from the previous product) = 79 thousands, + 7 tens times 3 hundreds, or 21 thousands, = 100 thousands = 10 ten-thousands. We write 0 in the thousands' place of the product. 7 tens times 8 thousands = 56 ten-thousands, + 10 ten-thousands (from the previous product) = 66 ten-thousands, which we write. The multiplication being now completed shows the answer to be 660124.

The following will exhibit the method to the eye: —

| | | |
|---:|---|---|
| 8356. | | |
| 79. | | |
| 54 | units | = 9 × 6. |
| 45 | tens | = 9 × 50. |
| 42 | tens | = 70 × 6. |
| 27 | hundreds | = 9 × 300. |
| 35 | hundreds | = 70 × 50. |
| 72 | thousands | = 9 × 8000. |
| 21 | thousands | = 70 × 300. |
| 56 | ten-thousands | = 70 × 8000. |
| 660,124 | | = 79 × 8356. |

When the pupil clearly understands how the different denominations of the product are obtained, it will be well to shorten the work somewhat by omitting to name the denomination of the factors used: —

Thus, 9 × 6 = 54 units = 5 tens and 4 units.

9 × 5 = 45, + 5 = 50, + 7 × 6, or 42, = 92. 92 tens = 9 hundreds and 2 tens.

9 × 3 = 27, + 9 = 36, + 7 × 5, or 35, = 71. 71 hundreds = 7 thousands and 1 hundred.

9 × 8 = 72, + 7 = 79, + 7 × 3, or 21, = 100. 100 thousands = 10 ten-thousands and 0 thousands.

7 × 8 = 56, + 10 = 66. 66 ten-thousands.

This gives for an answer 660124, as did the first method. The last process being understood, the work may be still further abbreviated by omitting to name the factors used.

Thus, 54 units = 5 tens and 4 units.

45 + 5 = 50, + 42 = 92. 92 tens = 9 hundreds and 2 tens.

27 + 9 = 36, + 35 = 71. 71 hundreds = 7 hundreds and 1 thousand.

72 + 7 = 79, + 21 = 100. 100 thousands = 10 ten-thousands and 0 thousands.

56 + 10 = 66. 66 ten-thousands.

Answer, as before, 660124.

Finally, the work may be abbreviated so as to name only results: —

54 units = 5 tens and 4 units.

45, 50, 92 tens = 9 hundreds and 3 tens.

27, 36, 71 hundreds = 7 thousands and 1 hundred.

72, 79, 100 thousands = 10 ten-thousands and 0 thousands.

56, 66 ten-thousands.

**Note.** The above methods are much more expeditious than is the method of writing the product by each figure of the multiplier separately, and are no more liable to inaccuracy.

1. How much will 97 acres of land cost at $347 per acre?

2. If a cubic yard of sand weighs 2537 lb., how much will 38 cubic yards weigh?

3. How many pounds are there in 18 T. 17 cwt. 1 qr.?

4. If a ship sails 96 miles in one day, how far will she sail in 247 days?

5. Bought 24 bundles of hay, each bundle containing 497 lb. How many pounds were there in all?

6. Bought 2947 gallons of oil at $.84 per gallon. How much did it cost? Sold it for $.97 per gallon. How much was received for it? What was the gain on it?

7. Mr. Russell bought 86 balls of twine, each ball containing

8794 ft., and Mr. Greene bought 5.7 times as much. How many feet of twine did Mr. Russell buy? How many did Mr. Greene buy?

8. How much will 8.3 casks of old wine cost at $138.47 per cask?

9. How much will .67 of a ton of lead cost at $139.48 per ton?

10. Mr. Hovey bought 6247 feet of land, and Mr. Ewell bought .94 of the quantity. How many feet did Mr. Ewell buy?

11. How many pounds are there in 958 boxes of sugar, each box containing 743.67 lb.?

NOTE. The products and sums employed in solving the above example are given below, but the pupil should be prepared to give a more full explanation.

56 hundredths = 5 tenths and 6 hundredths.

5 + 48 + 35 = 88. 88 tenths = 8 units and 8 tenths.

8 + 24 + 30 + 63 = 125. 125 units = 12 tens and 5 units.

12 + 32 + 15 + 54 = 113. 113 tens = 11 hundreds and 3 tens.

11 + 56 + 20 + 27 = 114. 114 hundreds = 11 thousands and 4 hundreds.

11 + 35 + 36 = 82. 82 thousands = 8 ten-thousands and 2 thousands.

8 + 63 = 71. 71 ten-thousands.

The answer, therefore, is 712435.86 lb.

12. How many are 8795 times 96543?

NOTE. By extending the principles before explained we can write the final product at once, as below.

96543.
8795.
849095685.

In the following forms, 2 products are written:—

96543.
8795.
9171585 units = 95 × 96543.
8399241 hundreds = 8700 × 96543.
849095685. = 8795 × 96543.

$$\begin{array}{r l l}
96543. & & \\
8795. & & \\
\hline
76751685 & \text{units} & = 795 \times 96543. \\
772344\phantom{000} & \text{thousands} & = 8000 \times 96543. \\
\hline
849095685. & & = 8795 \times 96543.
\end{array}$$

13. If a cubic foot of iron weighs 486.25 lb., how much will 347 cubic ft. weigh?

14. If 1 cubic foot of walnut timber weighs 41.9375 lb., how much will 43 sticks of hewn walnut timber, each 16 ft. long, 1½ ft. wide, and 1 ft. thick, weigh?

15. How many square feet are there in a rectangular lot, 4327 feet long and 249 ft. wide?

16. How much will 354.87 acres of land cost at $83.968 per acre?

17. What will 3.798 tons of hay cost at $14.278 per ton?

18. I bought 287 bales of cloth, each bale containing 247.986 yards. How many yards did they all contain?

19. If a horse trots 3.674 hours, at the average rate of 7.2968 miles per hour, how far will he trot?

20. How many square inches are there in a lot 247 ft. long and 187 ft. wide?

21. What will 47.983 yards of cloth cost at $2.83 per yd.?

22. What will 678.94 bbl. of flour cost at $6.37 per bbl.?

23. How many solid inches in 5 C. 6 Cd. ft. 12 cu. ft. 1437 cu. in.?

24. How many sq. in. in 8 sq. rd. 21 sq. yd. 6 sq. ft. and 28 sq. in.?

25. How many dr. in 18 T. 16 cwt. 1 qr. 14 lb. 6 oz. 11 dr.?

26. A grain dealer sold 28.7 bushels of wheat at $1.294 per bushel, and 14.79 bushels at $1.267 per bu. What did he receive for it?

27. A city merchant went into the country to purchase flour. He was absent from the city 27 days, and his expenses while absent were $7.386 per day. He bought 175 bbl. of flour at $5.875 per bbl., 516 bbl. at $5.948 per bbl., 1386 bbl. at $6.11 per bbl., and 827 bbl. at $6.087 per bbl. It cost him $.634 per bbl. to get the flour transported to the city. He sold 697 bbl. of it at $7.114, 824 bbl. at $7.213 per bbl., and the rest at $6.978 per bbl. Did he gain or lose by the adventure, and how much?

28. A merchant bought 49.5 cases of cassimere, each case

containing 297 yd. at \$1.1875 per yd. It cost him \$.125 per case to have the cloth removed to his store, and \$.045 per case to have it hoisted into his loft. One case of the cloth was stolen from him; he sold 23 cases at \$1.423 per yd., and the remainder at \$1.357 per yard, agreeing to deliver it at a railroad depot, 1 mile from his store. It cost him \$.158 per case to have it carried to the depot. Did he gain or lose on the cloth, and how much?

## SECTION XXIII.

**A.** 1. How many apples, at 3 cents apiece, can be bought for 24 cents? This is a question in Division. It is obvious that, to determine its answer, we must find how many parts of 3 cents each there are in 24 cents; for each such part is the price of 1 apple.

We can find the number of these parts by counting 24 cents into piles of 3 cents each, and then counting the number of piles; — by finding how many threes must be added to make 24; — or by finding how many times 3 is contained in 24, as we have done in previous examples of this character. This last method is called Division.

2. If 3 pounds of raisins cost 24 cents, how many cents will 1 lb. cost? This is also a question in Division. To determine its answer, we must find how many cents there will be in each part obtained by separating 24 cents into 3 equal parts.

We can do this by laying out 24 cents into 3 equal piles, and then counting the number of cents in each pile; — or by finding $\frac{1}{3}$ of 24 in the usual manner. This last would be called the process of division.

All questions in division require results similar to that required by one or the other of the above. We may, therefore, give the following definition :—

Division is a process by which we ascertain the number of parts of a given size into which a given number may be separated, or by which we ascertain the number of units there will be in each part obtained by dividing a given number into a given number of equal parts.

The nature of the question is the only thing that will determine which of these results is required. The same arithmetical process may be employed to obtain the answer to a question of either kind, but the reasoning process required in one is very different from that required in the other. The following solutions to questions 1 and 2 will illustrate the resemblances and differences above alluded to.

*Ques.* 1. *Solution.* If for 3 cents one apple can be bought, for 24 cents as many apples can be bought as there are times 3 cents in 24 cents, which are 8 times. Therefore, 8 apples can be bought for 24 cents, when one apple costs 3 cents.

*Ques.* 2. *Solution.* If 3 pounds of raisins cost 24 cents, one pound will cost one third of 24 cents, which is 8 cents. Therefore, one pound of raisins will cost 8 cents, when 3 pounds cost 24 cents.

The number supposed to be divided into parts is called the *dividend*. In the first class of questions, the number which shows the size of each part, and, in the second class, that which shows the number of parts, is called the *divisor*. The result is called the *quotient*.

We perform the division and make the necessary reductions in the same way, whether we write our work or not. If we write it, when our divisor is a small number, we usually write only the divisor, dividend, quotient, figure by figure as we obtain it, and the final remainder, if there be one; but when the divisor is a large number, we usually write the divisor, dividend, and quotient, as before, and also the products of the divisor by each figure of the quotient, with the remainder obtained by subtracting these products from the corresponding denominations of the dividend.

**B.** To illustrate the usual method of writing the work when the divisor is a small number, we give the following example: —

How many barrels of flour, at 8 dollars per barrel, can be bought for 29576 dollars?

By the usual reasoning process we determine that as many barrels can be bought as there are parts of 8 dollars each in 29576 dollars.

$$\begin{array}{rl} & \text{Dividend.} \\ \text{Divisor. } \$8.\ ) & \underline{\$29576.} \\ \text{Quotient.} & \ \ 3697. \end{array}$$

= number of parts of \$8 each = number of barrels that can be bought.

In explaining the division, we should say, As in 2 ten-thousands there are not as many as ten thousand parts of 8 each, we ascertain how many thousand parts of 8 each there are in the 29 thousands. As 8 is contained in 29 thousands 3 thousand times, with a remainder of 5 thousands, we write 3 in the thousands' place of the quotient, and reduce the 5 thousands to hundreds, adding to them the 5 hundreds in the hundreds' place of the dividend. This gives 55 hundreds.

As 8 is contained in 55 hundreds 6 hundred times, with a remainder of 7 hundreds, we write 6 in the hundreds' place of the quotient, and reduce the 7 hundreds to tens, adding to them the 7 tens written in the tens' place of the dividend. This gives 77 tens.

As 8 is contained in 77 tens 9 tens times, with a remainder of 5 tens, we write 9 in the tens' place of the quotient, and reduce the 5 tens to units, adding them to the 6 units written in the units' place. This gives 56 units.

As 8 is contained in 56 units 7 times, with no remainder, we write 7 in the units' place of the quotient.

We thus find that the quotient is 3697, which represents the number of parts of $8 each there are in the dividend, and since each such part is the price of 1 barrel, the number of barrels that can be bought must equal the number of these parts.

To test the correctness of the previous work, we have only to ascertain whether 3697 times 8 dollars will equal 29576 dollars. For, since 1 barrel costs $8, 3697 barrels must cost 3697 times $8, which ought to be $29576.

1. How many yards of German broadcloth at $6 per yard can be bought for $174564?

2. How many sheep at $7 apiece can be bought for $7833?

3. If 8 melons cost $1, how many dollars will 3744 melons cost?

4. How many weeks are equal to 87638 days?

5. How many feet are equal to 69847 inches?

6. How many ounces are equal to 98764 ʒ?

7. How many square yards are equal to 5987536 sq. ft.?

8. How many yards and quarters are equal to 97875 qr.?

9. How many bu., pk. and qt. are equal to 87637 qt.?

10. How many gal., qt., pt. and gi. are equal to 798953 gills?

11. How many ℔, ℥, ʒ, and ℈ are equal to 793527 ℈?

12. How many £, s., d. and qr. are equal to 587537qr.?

13. How many sq. yd. in a surface 984 ft. long and 876 ft wide?

14. In a cornfield there are 488 rows and 928 hills in a row, each hill producing a half pint of corn. How many half pints are there in the field? How many quarts? How many pecks? How many bushels?

15. A man bought some land, paying for it at the rate of $1 for every 7 square rods. The field he bought was 1673 rods long, and 985 rods wide. How much did it cost him? He paid for it in half-eagles. How many did it take?

16. Reduce $\frac{978435}{9}$ to ones.

17. Reduce $\frac{86379}{7}$ to ones.

18. Reduce $\frac{583793}{8}$ to ones.

19. Reduce $\frac{748395}{3}$ to ones.

20. Reduce $\frac{894732}{6}$ to ones.

**C.** 1. If 8 acres of land cost $9707, what will 1 acre cost? By the usual reasoning process we determine that 1 acre will cost as many dollars as there will be in each part obtained by dividing $9707 into 8 equal parts, that is, each acre will cost $\frac{1}{8}$ of $9707.

$$\begin{array}{rll} \text{Divisor. } 8\,) & \underline{\$9707,} & \text{Dividend} = \text{cost of 8 acres.} \\ & 1213\frac{3}{8}, & \text{Quotient} = \text{cost of 1 acre.} \end{array}$$

We divide thus: — $\frac{1}{8}$ of 9 thousands = 1 thousand, with a remainder of 1 thousand. 1 thousand = 10 hundreds, and 7 hundreds added = 17 hundreds. $\frac{1}{8}$ of 17 hundreds = 2 hundreds, with a remainder of 1 hundred. 1 hundred = 10 tens, and there are no tens to add. $\frac{1}{8}$ of 10 tens = 1 ten, with a remainder of 2 tens. 2 tens = 20 units, and 7 units added = 27 units. $\frac{1}{8}$ of 27 units = 3 units, with a remainder of 3 units. $\frac{1}{8}$ of 3 units = $\frac{3}{8}$ of a unit. Therefore, $\frac{1}{8}$ of $9707 = $1213$\frac{3}{8}$, and hence 1 acre will cost $1213$\frac{3}{8}$ when 8 acres cost $9707.

To prove this work we can ascertain if 8 times $1213$\frac{3}{8}$ are equal to $9707. For, if 1 acre costs $1213$\frac{3}{8}$, 8 acres will cost 8 times $1213$\frac{3}{8}$, which ought to be $9707.

We might have obtained the answer in another form, by reducing the 3 units, remaining from the division of the units, to lower denominations, thus: — 3 units = 30 tenths, and $\frac{1}{8}$ of 30 tenths = 3 tenths, with a remainder of 6 tenths. 6 tenths = 60 hundredths, and $\frac{1}{8}$ of 60 hundredths = 7 hundredths, with a remainder of 4 hundredths. 4 hundredths = 40 thousandths, and $\frac{1}{8}$ of 40 thousandths = 5 thousandths. The answer now appears in the form, $1213.375, which may be proved to be correct in the same way that the first form was proved.

The second form of answer might have been obtained from the first by reducing the $\frac{3}{8}$ of a dollar to a decimal fraction.

2. If 7 cases of cloth cost $6545, how many dollars will 1 case cost?

3. If 18579 lb. of hay are obtained from 9 acres, how many pounds are obtained from 1 acre?

4. If 7 horses can draw 16786 lb., how many pounds can 1 horse draw?

5. If it cost $173549 to build 9 miles of rail-road, how much will it cost to build 1 mile?

6. A father divided his estate, valued at $879643, equally among his 5 children. What was the share of each?

7. What is $\frac{1}{7}$ of 975646?

8. What is $\frac{3}{8}$ of 3954.7 bushels?

9. What is $\frac{5}{8}$ of 7354.278 tons?

10. What is $\frac{5}{6}$ of 12456.78 miles?

11. If 6 piano fortes cost $1837.44, how much will 7 cost?

12. The ship Surprise is valued at $25427. Joseph Ward owns $\frac{2}{5}$ of her, and Samuel Lowe owns the remainder. What is the value of Mr. Ward's share? of Mr Lowe's?

13. A father, dying, left to his son an estate valued at $87646.44. The first year after the son came into possession of the property he spent $\frac{1}{7}$ of it in dissipation, the second year he spent $\frac{2}{3}$ of the remainder, and at the end of the third year he was penniless. How much did he spend in the first year? How much in the second year? How much in the third?

14. Statistics show that the whole number of persons relieved or supported as paupers in Massachusetts during the year 1849, was 24892, and that 141 less than $\frac{4}{7}$ of this number were state paupers, and the remainder were town paupers. How many were state paupers? How many were town paupers? 309$\frac{1}{4}$ less than $\frac{3}{4}$ of the state paupers were foreigners. What was the number of foreigners? 804 more than $\frac{1}{2}$ of the whole were made paupers by intemperance in themselves or others. What was the number of these?

15. A benevolent society divided 10 cwt. 3 qr. 23 lb. of flour equally among 6 poor persons. What was each person's share?

*Solution.* If 6 persons had 10 cwt. 3 qr. 23 lb. of flour, 1 person would have $\frac{1}{6}$ of 10 cwt. 3 qr. 23 lb.

$$\begin{array}{r|llll l}
 & \text{cwt.} & \text{qr.} & \text{lb.} & & \\
6\,) & 10, & 3, & 23. & & = \text{share of 6 persons.} \\
\hline
 & & & & \text{oz.} & \\
 & 1, & 3, & 8, & 8. & = \text{share of 1 person.}
\end{array}$$

*Explanation of process.* $\frac{1}{6}$ of 10 cwt. is 1 cwt. with a remainder of 4 cwt. 4 cwt. = 16 qr., and 3 qr. added = 19 qr. $\frac{1}{6}$ of 19 qr. = 3 qr. with a remainder of 1 qr. 1 qr. = 28 lb., and 23 lb. added = 51 lb. $\frac{1}{6}$ of 51 lb. = 8 lb. with a remainder of 3 lb. 3 lb. = 48 oz. $\frac{1}{6}$ of 48 oz. = 8 oz.

The quotient is, therefore, 1 cwt. 3 qr. 8 lb. 8 oz.

16. 10 men bought a stack of hay weighing 4 T. 15 cwt. 3 qr. 26 lb. 14 oz. 6 dr. What ought each man's share to weigh?

17. A farmer had 9 equal bins filled with grain. They all contained 2047 cu. ft. 1436 cu. in. How many feet and inches did each bin contain?

18. The brig Maria sailed from Philadelphia to Boston, with 207 T. 13 cwt. 2 qr. 19 lb. of coal, $\frac{5}{7}$ of which was landed at one wharf, and the remainder at another. What was the weight of that landed at the first wharf? at the second?

19. If John walks $\frac{5}{7}$ as fast as George, how far will John walk while George is walking 97 m. 6 fur. 37 rd. 2 yd.?

20. A company of 9 California gold-diggers found in 1 month 37 lb. 4 oz. 16 dwt. 17 gr. of gold, which they divided equally. What was the share of each?

21. An English ship, valued at 8743£, carries a cargo which is worth $\frac{7}{8}$ as much as the ship. How many £, s. and d. is the cargo worth? In a storm the sailors were obliged to throw $\frac{1}{4}$ of the cargo overboard. What was the value of what they threw overboard?

22. A merchant bought 17 T. 14 cwt. 2 qr. 23 lb. 10 oz. of sugar. He sold $\frac{2}{7}$ of it to one man, $\frac{3}{5}$ of the remainder to another, and what there was left to another. What was the weight of that which he sold to the first man? to the second? to the third?

**D.** The following illustrates the method of dividing, when, in the first class of questions, there are decimal fractions in the divisor.

1. **How many baskets, each containing .8 of a bushel, can be filled from 347.54 bushels of peaches?**

WRITTEN WORK.

347.54 = number of bushels of peaches.

347.54 = number of baskets holding 1 bushel each.
10 times 347.54 = 3475.4 = number of baskets holding .1 of a bu. each.
$\frac{1}{8}$ of 3475.4 = 434.425 = number of baskets holding .8 of a bu. each.

*Explanation.* If each basket held 1 bushel, there would be as many baskets required as there are bushels, or 347.54 baskets. If each basket held .1 of a bushel, it would take 10 baskets to hold 1 bushel, and hence, there would be 10 times as many baskets as there are bushels, or 3475.4 baskets. But, when each basket holds .8 of a bushel, it will take $\frac{1}{8}$ as many as when each basket holds .1 of a bushel, or $\frac{1}{8}$ of 3475.4 baskets, which, found by the methods already explained, is 434.425 baskets.

2. How many vest patterns, each containing .9 of a yard in a piece of cloth containing 34.758 yds.?

3. How many lots, each containing .7 of an acre, can be made from a field containing 94.381 acres?

4. How many pounds of nails, at \$.09 per lb., can be bought for \$94.734?

5. How many pounds of cheese, at \$.08 per lb., can be bought for \$7.50?

6. How many yards of sheeting, at \$.08 per yd., can be bought for \$273?

7. If .04 of a yard of broadcloth is worth 1 yd. of calico, how many yards of calico are 85.4784 yd. of broadcloth worth?

8. If .008 of a yard of silk is worth 1 yard of tape, how many yards of tape are 375 yd. of silk worth?

The following example and explanation illustrate the method of dividing when in the second class of questions the divisor contains decimal fractions.

9. If .06 of a lot of land cost \$1847.52, how much will the whole lot cost?

WRITTEN WORK.

\$1847.52 = cost of .06 of the lot.

$\frac{1}{6}$ of \$1847.52 = \$307.92 = cost of .01 of the lot.
100 times \$307.92 = \$30792. = cost of the lot.

*Explanation.* If .06 of the lot cost \$1847.52, .01 of the lot will cost $\frac{1}{6}$ of \$1847.52, which, found by the method before explained, is $307.92. If .01 of the lot costs $307.92, the whole lot will cost 100 times $307.92, which will be found by removing the point two places to the right.

10. If .07 of a cask of oil cost $2.5739, what will the whole cask cost?

11. If .9 of a chest of tea cost $9.378, what will a chest of the same kind of tea cost?

12. Mr. Smith borrowed some money, agreeing to pay interest on it at the rate of 6 per cent per year. He found that the yearly interest of the sum he had borrowed was $47.64. How much did he borrow?

13. Mr. Abbot lent a large sum of money, on which he was to receive interest at the rate of 6 per cent per year. The interest of it for 6 months was $173.58. How much money did he lend?

14. The taxes on property in a certain town are .004 of all the property taxed. How much money ought a person to be worth, who pays a property tax of $18.75?

15. 5238.852 = $\frac{6}{7}$ of what number?

16. 7948.458 = .6 of what number?

17. A farmer, on measuring the grain he had raised, found that he had .7 as much rye as oats, .9 as much oats as barley, and .8 as much barley as Indian corn. He had 85.05 bushels of rye. How many bushels of oats had he? of barley? of Indian corn?

18. If .5 of a bag of coffee costs $11.737, and .3 of a chest of tea costs $17.862, how much will 1 bag of coffee and 1 chest of tea cost? How much will 4 bags of coffee and 3 chests of tea cost?

19. If .04 of the value of my farm is $27.89, what is the value of my neighbor's farm, which is worth 3 times as much as mine?

20. .17 of my property is invested in rail-road stock, .57 of it in bank stock, .18 of it in real estate, and the rest, $475, is at interest. How much property have I? What is the value of my bank stock? of my rail-road stock? of my real estate?

**E.** When the divisor is the product of factors, it will sometimes be convenient to divide by its factors, instead of dividing by the whole number at once. The true quotient will be obtained by dividing the dividend by one factor, the quotient of this division by another, the quotient of the last division by another, and so on, till all the factors of the set considered are used.

1. How many days are there in 2375 hours?

To solve this question, we must find how many parts of 24 hours each there are in 2375 hours. Since 24 = 8 times 3, we may first find how many parts of 3 hours each there are, by dividing by 3 in the usual manner. There are 791 such parts with a remainder of 2 hours. But for every 8 parts of 3 hours each, there will be 24 hours. There will be as many parts of 24 hours each as there are times 8 parts of 3 hours each. By dividing 791 by 8 we find that there are 98 parts of 24 hours each, with a remainder of 7 parts of 3 hours each. There are, therefore, 98 days, and since we had by the first division a remainder of 2 hours, and by the second a remainder of 7 parts of 3 hours each, by both we had a remainder equal to $2 + 7 \times 3$ hours, or 23 hours. The answer is, therefore, 98 days and 23 hours, or $98\frac{23}{24}$ days.

WRITTEN WORK.

3 ) 2375 hours — 2 hours, 1st remainder.
8 ) 791 parts of 3 h. each — 7 parts of 3 h. each, 2nd remainder.
98 parts of 24 h. each = 98 days.

Final remainder, 2 h. + 7 parts of 3 h. each = 23 hours.

*Ans.* 98 da. 23 h., or $98\frac{23}{24}$ da.

2. How many pennyweights are there in 26371 grains?

3. How many quarters are there in 7277 pounds?

4. How many cubic yards are there in 276533 cu. ft.?

5. How many cord ft. are there in 149551 cu. ft.?

6. How many bins, each containing 36 bushels, can be filled from 31479 bushels of grain?

7. How many chests containing 25 lb. each, are required to hold 20673 lb. of tea?

8. If 1 acre of land can be bought for $54, how many acres can be bought for $47456?

9. If 1 ton of iron can be bought for $64, how many tons can be bought for $15151?

10. If 56 lb. of chalk cost 1 dollar, how many dollars will 272989 lb. cost?

11. A person who had 2159 marbles wished to divide them equally among 48 boys. How many marbles could he give to each boy?

To solve this question, we must find how many marbles there will be in each part obtained by dividing 2159 marbles into 48 equal parts. Since 48 = 6 times 8, we may first find how many marbles there would be in each part obtained by dividing 2159 marbles into 8 equal parts; *i. e.*, how many marbles there are in $\frac{1}{8}$ of 2159. There would be 269 marbles in each part, and 7 marbles remaining undivided. Now, if one of the 8 parts of 269 marbles each, were divided into 6 other parts, it is evident that each part thus obtained would contain as many marbles as there would be in $\frac{1}{48}$ of the original dividend. By dividing, we find that 269 marbles equal 6 parts of 44 marbles each, with a remainder of 5 marbles. Therefore, there will be 44 marbles in each part obtained by dividing 2159 marbles into 48 equal parts. To obtain the true remainder, observe that by the first division there was a remainder of 7 marbles, and that since one of the parts of 269 each gives a remainder of 5, 8 of them must give a remainder of 8 times 5, or 40 marbles, which, added to the 7 from the first division, gives 47 as the true remainder. Therefore, he could give 44 marbles to each boy, and have 47 marbles remaining.

### WRITTEN WORK.

8 ) 2159 marbles to be divided — 7 marbles, 1st remainder.
6 ) 269 { marbles in each of 8 parts — 5 marbles, remainder on each part.
44 marbles in each of 48 parts = share of each boy.

Final remainder = 7 + 5 on each of 8 parts = 7 + 40 = 47 marbles.

*Ans.* Each boy could have 44 marbles, and there would be 47 marbles remaining undivided.

12. A drover had 2644 sheep, which he wished to sell in 27 equal lots. How many sheep could he put in each lot?

13. A general wished to form his army of 3487 men into 36 equal companies. How many men could he put in each company?

14. How many eagles will there be in each part obtained by dividing 9785 eagles into 18 equal parts?

15. What is $\frac{1}{63}$ of 6925? $\frac{1}{42}$ of 9539?

16. What is $\frac{1}{81}$ of 74853? $\frac{1}{35}$ of 6984?

17. If 53469 books are divided equally among 54 persons what will be the share of each?

18. A drover wishes to sell a drove of 2055 sheep in 21 days, by selling the same number each day. How many should he

sell each day to come as near as possible to selling out, and how many sheep will remain unsold at the end of the time?

**F.** From what we have already learned of the nature of division, it is evident that, where there is no remainder, the divisor multiplied by the quotient must produce the dividend; and that the dividend divided by the quotient must produce the divisor. When there is a remainder, it is equally evident that if the divisor and quotient be multiplied together, and the remainder be added to their product, the result will equal the dividend. If the remainder be subtracted from the dividend, and the remainder thus obtained be divided by the divisor, the result will equal the quotient; or, if it be divided by the quotient, the result will equal the divisor.

The remainder should always be less than the divisor; for, if it is not, the dividend will contain the divisor more times than is indicated by the quotient figure.

When the divisor is a large number, we cannot be certain that we have obtained a correct quotient figure till we have found the product of the divisor by the quotient, and the remainder after subtracting this product from the dividend.

The product of the divisor by the quotient should either be equal to, or less than, the dividend. When this product is equal to the dividend, the division can be exactly performed, but when it is less, there is a remainder, which may be found by subtracting it from the dividend. If, in any case, the product of the divisor by the supposed quotient is greater than the dividend, it shows that the divisor is not contained as many times in the dividend as the supposed quotient indicates.

We will now apply these principles to a few examples.

1. How many lots of 53 acres each can be made from 424 acres?

*To Teacher.* In this and all subsequent examples, the pupil should give the reasoning process, and tell whether the example requires the number of parts of a given size in the dividend, or the size of each part when the number of parts is given.

acres. acres.
53 ) 424 ( 8 parts of 53 acres each = number of lots.
424 = Product of divisor by quotient.

We cannot tell at once how many times 53 there are in 424, but since 53 differs but little from 5 tens, we will use 5 for a trial divisor. As 5 is contained 8 times in 42, with a remainder, we may infer that 53 is contained 8 times in 424. To see whether this be correct or not, we multiply 53 by 8, and as the product is 424, it follows that 53 is contained exactly 8 times in 424, and, therefore, that 8 lots can be made under the given conditions.

2. How many hogsheads, each containing 63 gallons, can be filled from 556 gallons of vinegar?

*Explanation.* Making 6 our trial divisor, we observe that in 55 it is contained 9 times, with a remainder; we, therefore, make 9 the trial quotient of the division of 656 by 63, and, to test whether our supposition be correct, we multiply 63 by 9. The product is 567, which is more than 556; therefore, 63 cannot be contained as many as 9 times in 556. We must now try for a quotient a number less than 9, and, as 9 times our divisor differed from the dividend by less than once the divisor, we infer that 8 is the true quotient figure. Multiplying the divisor by 8, we find for a product 504, which, being less than our dividend, shows that 8 is not too large. Subtracting 504 from 556 we have a remainder of 52 gallons, which shows us that 556 gallons will fill 8 hogsheads of 63 gallons each, and that there will be a remainder of 52 gallons, which would fill $\frac{52}{63}$ of a hhd. If we should subtract 52 gallons from 556 gallons the remainder ought to fill just 8 hogsheads, and this suggests some methods of proof.

WRITTEN WORK.

gal. gal. quotient.
63 ) 556 ( 8 parts of 63 gallons each.
504 = 8 times 63.
52 gallons remainder.

*Ans.* 8 hhd. 52 gal., or $8\frac{52}{63}$ hhd.

3. How many stacks of hay, each stack weighing 8743 pounds, can be made from 57617 pounds of hay?

lb. lb.
8743 ) 57617 ( 6
52458
5159

In performing the necessary division observe that as the divisor is nearer 9 thousands than 8 thousands, we shall be likely to come nearer the true quotient by taking 9, than by taking 8 as the trial divisor. As 9 is contained 6 times with a remainder in 57, we infer that 6 will be the true quotient. We find that 6 parts of 8743 pounds each = 52458 pounds, which being less than the dividend, shows that the quotient figure is not too large. Subtracting this product, the remainder is 5159 pounds, which, being less than the divisor, shows that the quotient figure is correct. Therefore, by the conditions of the question, 6 stacks can be made, and there will remain 5159 pounds of hay unused, which might have been taken from the whole quantity without affecting the number of stacks of 8743 lb. each.

4. In a pasture there are 191 sheep, which their owner wishes to divide equally among 26 farmers. How many can he give to each farmer?

In performing the necessary division in this example, observe that as the divisor, 26, is nearer 3 tens than 2 tens, we shall be more likely to obtain the true quotient by taking 3 as the trial divisor, than by taking 2. 3 being contained 6 times with a remainder in 19, we try 6 for a quotient. But 26 parts of 6 each, give 156 for a product, which, subtracted from the dividend, leaves a remainder of 35. As the remainder is greater than the divisor, the trial quotient must be too small. We must, therefore, try a larger number for a quotient. Inspecting the remainder we see that it can contain the divisor but once, with a remainder of 9. Therefore, the dividend will contain the divisor one more than 6 times, which will be 7 times, and there will be a remainder of 9; that is, each man could take 7 sheep, and there would be 9 left that could not be distributed without giving to one man more than to another.

The following examples are to be performed in a similar manner.

5. How many rooms can be carpeted by 237 yd. of carpeting, if each room can be carpeted by 24 yd.?

6. How many gold watches, at $87 apiece, can be bought for $613?

7. How many dollars will 448 oranges cost, if 56 oranges cost 1 dollar?

8. How many square feet are there in 1327 inches?

9. How many cubic feet in 12346 cubic inches?

10. $\frac{479}{54}$ = how many ones?

11. $\frac{1987}{294}$ = how many ones?

12. $\frac{8578}{1637}$ = how many ones?

13. $\frac{94376}{23572}$ = how many ones?

14. $\frac{578639}{87426}$ = how many ones?

15. If 495 men consume 2367 lb. of meat in 1 week, how many pounds will 1 man consume?

16. If 384 kegs will contain 3456 gallons, how many gallons will 1 keg contain?

17. If 7427 barrels of flour can be bought for 69359 dollars, for how many dollars can 1 barrel be bought?

18. If 37 lb. of rice cost $1.48, how much will 1 lb. cost?

19. If 67 yd. of cotton cloth cost $6.03, how much will 1 yd. cost?

**G.** 1. A miller has 9233 lb. of flour, which he wishes to put into bags, each containing 28 lb. How many bags can he fill?

*Solution.* If 28 lb. will fill 1 bag, 9233 lb. will fill as many bags as there are parts of 28 lb. each in 9233 lb., to ascertain which we must divide 9233 by 28.

The following represents a convenient way of writing the figures used in performing the work: —

div'r. divi'd. quotient.
28 ) 9233 ( 329
84 hundreds = 300 × 28.
83 tens.
56 tens = 20 × 28.
273 units.
252 units = 9 × 28.
21 units = remainder.

*Ans.* 329 bags, with 21 pounds remaining unused, or $329\frac{21}{28}$ bags.

*Explanation.* The divisor being nearer 3 tens than 2 tens, 3 will be a more convenient trial divisor than 2. Since 3 is contained 3 times in 9, we infer that 28 is contained 3 hundred times in 92 hundreds. Multiplying 28 by 3, placing the product under the hundreds of the dividend to mark its denomination, (which is equivalent to multiplying 28 by 300,) we have 84 hundreds, which, being less than the part of the dividend we consider, shows that the trial quotient figure is not too large. Subtracting this product from the hundreds of the dividend, we find a remainder of 8 hundreds, which being less than 2800 shows that the trial quotient figure is the true one. 8 hundreds = 80 tens, to which adding the 3 tens of the dividend, we have 83 tens to be divided by 28. Since the trial divisor, 3, is contained twice in 8, we infer that 28 is contained 2 tens times in 83 tens. 2 tens, or 20, times 28 = 56 tens, which, being less than 83 tens, shows that the trial quotient figure is not too large. 56 tens from 83 tens = 27 tens, which, being less than 28 tens, shows that the trial quotient figure is the true one. 27 tens = 270 units, to which adding the 3 units in the units' place of the dividend, we have 273 to be divided by 28. Since the trial divisor, 3, is contained 9 times in 27, we infer that 28 is contained 9 times in 273. 9 times 28 = 252, which being less than 273 shows that the trial quotient is not too large. 252 from 273 = 21, which, being less than 28, shows that the trial quotient figure of the units is the true one. The entire quotient is, therefore, 329, and the remainder is 21, thus showing that the miller could fill 329 bags, and have 21 lb. of flour left; or, that he could fill 329$\frac{21}{28}$ bags.

The answer might have been obtained in another form by reducing the remainder, 21 units, to lower denominations and continuing the division as follows:—

21 units = 210 tenths. Since 3 is contained 7 times in 21, we infer that 28 is contained 7 tenths times in 210 tenths. 7 tenths times 28 = 196 tenths, which, being less than 210 tenths, shows that the trial quotient figure is not too large. Subtracting as we have hitherto done gives a remainder of 14 tenths, which is equal to 140 hundredths. The trial divisor 3 is contained almost 5 times in 14, and as it is larger than the tens of the true divisor, it is probable that the true divisor will be contained .05 times in 140 hundredths. But .05 times 28 = 140 hundredths. The quotient is, therefore, 329.75. This form might have been obtained by reducing $\frac{21}{28}$ to a decimal fraction.

By the previous illustrations, it appears that when the divisor is a large number, we proceed as follows:—

1st. Find a convenient trial divisor.

2d. With the trial divisor, determine the probable quotient figure of the highest denomination.

3d. Find the product of the divisor by this figure.

4th. Compare this product with the part of the dividend considered, to see that the quotient figure be not too large.

(If it should be found to be too large, a smaller one must be substituted for it, and the product of the divisor by the smaller figure must be substituted for the former product.)

5th. Subtract the product from the part of the dividend considered.

6th. Compare this remainder with the divisor, to see that the quotient figure is not too small.

(The quotient figure will be too small whenever the remainder is larger than the divisor. In that case, the quotient figure, product, and remainder should be removed, and a larger quotient figure should be substituted with its appropriate product and remainder.)

7th. If all the denominations of the dividend have not been considered, reduce the remainder to the next lower denomination, and add to it the figure of that denomination in the dividend.

8th. Proceed with this as before to find the next quotient figure, and so go on till all the denominations of the dividend have been considered, or, if decimal fractions are used in the quotient, till as many decimal places as are desirable have been obtained.

The position of the decimal point is to be determined in the same manner as when the divisor is a small number.

2. How many square feet are equal to 42912 square inches?

3. How many boxes containing 73 quarts each can be filled from 658764 quarts of meal?

4. How many watches at $48 each can be bought for $3456?

5. If a boy can perform 37 examples in one day, how many days will it take him to perform 8764 examples?

6. If a vessel sails 147 miles per day, how many days will it take her to sail 3579 miles?

7. If 252 pints of cider will fill a barrel, how many barrels can be filled from 876438 pints?

8. How many weights, each weighing 56 lb., will be required to balance 7 loads of iron, each weighing 9478 lb.?

9. James can pump 37 quarts of water per minute, and William can pump 43 quarts of water per minute. How many minutes will it take William to pump as much as James can pump in 473 minutes?

10. What will 1 acre of land cost if 237 acres cost $236526?

11. If 687 bushels of potatoes cost $515.25, what will 236 bushels cost?

12. If 67 yd. of silk cost $75.375, what will 48 yd. cost?

13. Gave 7 loads of hay at $17.48 per load, and 238 bushels of corn at $.67 per bushel, for 32 acres of land. How much money ought I to give for 7 acres of the same land?

14. If $\frac{28}{75}$ of a vessel is worth $6735.96, what is the whole vessel worth?

15. What will $\frac{87}{54}$ of a vessel cost, if the whole vessel costs $1874.34?

16. What is $\frac{63}{85}$ of $1976.43?

By performing the subtraction at the same time that we do the multiplication, we can avoid writing our products. Suppose that we wish to find the quotient of 1947.63 divided by 396. We will give the method, leaving the pupil to find the explanation of it. 396 is contained 4 times in 1947. 4 times 6 = 24, WRITTEN FIGURES.

```
396 ) 1947.63 ( 4.9182 quotient.
      363.6
       7.23
       3.270
        .1020
        .0228 remainder.
```

which subtracted from 27 leaves 3; this we write in the units' place. 4 times 9 = 36, and 2 added make 38, which subtracted from 44, leaves 6. 4 times 3 = 12, and 4 added make 16, which subtracted from 19 leaves 3. The first remainder is, therefore, 363. Changing this to tenths, and adding the 6 tenths, we have 3636 tenths, which divided by 396, gives 9 tenths for a quotient. We multiply and subtract as before, and thus proceed till the division is completed, or till we have as many decimal places in the quotient as we wish.

Perform the division in the following examples in the same way: —

17. What is the quotient or 78643 ÷ 47?

18. What is the quotient of 137648 ÷ 326?

19. What is $\frac{1}{37}$ of 65427?

20. What is $\frac{24}{579}$ of 3865726?

21. Reduce $\frac{16746}{86}$ to ones.

22. Reduce $\frac{27843}{144}$ to ones.

23. What is the value of 764379 cubic inches, expressed in cubic feet and inches?

24. How many square feet and inches are equal to 954376 sq. in.?

25. 4743864 = $\frac{72}{85}$ of what number?

26. 4743864 = .72 of what number?

27. If the wages of 96 laborers for 1 year are $26470.08, what will be the wages of 1 laborer for the same time?

28. If 875 pairs of boots cost $2737.16, how much will 1 pair cost?

29. At $.68 a pair, how many pairs of slippers can be bought for $97.58?

30. A man failing in business was able to pay $7578.38, which was only 37 per cent of his debts. What was the amount of his debts?

31. Mr. French spent $228.69 for tea, at $.27 per lb. He sold the tea for $.30 per lb., and with the money he received for

it he bought coffee at $.15 per lb. He sold $\frac{1}{2}$ of the coffee at $.16 per lb., and the rest for $.17 per lb. How much did he receive for the coffee?

32. When sugar was worth 9 cents per lb., a man gave 8766 lb. of it for 81 acres of land. How much did the land cost per acre?

33. If $\frac{14}{25}$ of a ton of hay cost as much as $\frac{18}{23}$ of a ton of straw, and 36 tons of straw cost $405.72, how much will a ton of hay cost?

34. A man sold a pile of wood, 96 ft. long, 8 ft. wide, and 7 ft. high, for $190.554. At what rate did he sell it per cord?

35. A furniture dealer bought 57 bedsteads at $4.73 apiece, which he sold so as to gain $44.64 by the transaction. He immediately invested the money he thus received for chairs at $.75 apiece. How many chairs did he buy?

36. A fruit dealer paid $169.75 for apples at $1.75 per barrel, $80.625 for apples at $1.875 per barrel, and $92.64 for apples at $1.93 per barrel. He sold them all at the rate of $2.068 per barrel. What did he receive for them? How much did he gain on them?

## SECTION XXIV.

It is frequently the case that by carefully examining the numbers we are to operate upon, we can discover some abbreviated method of performing the work. We have already suggested several such methods, and it is the design of this section to suggest others.

**A.** It is often advantageous to add on the principles involved in the following equations: —

1. 786 + 99 = 786 + 100 — 1.
2. 47834 + 57994 = 47834 + 58000 — 6.
3. 973876 + 49993 = 973876 + 50000 — 7.
4. 16£, 13s. 3d. + 7£, 19s. 8d. = 16£, 13s. 3d. + 8£, — 4d.
5. 14 ℔, 5 ℥, 2 ʒ 2 ℈, 13 gr. + 4 ℔, 11 ℥, 7 ʒ, 2 ℈, 16 gr. = 14 ℔, 5 ℥, 2 ʒ, 2 ℈, 13 gr. + 5 ℔, — 4 gr.
6. How many are 3476437 + 1499992?
7. How many are 732475 + 9999387?

8. What is the value of 48£, 13s. 5d. 2qr. + 99£, 19s. 11d. 1qr.?

9. What is the value of 8 cwt. 1 qr. 16 lb. 13 oz. 5 dr. + 5 cwt. 2 qr. 27 lb. 15 oz. 8 dr.?

10. What is the value of 19 rd. 4 yd. 2 ft. 4 in. + 18 rd. 5 yd. 0 ft. 6 in.?

**B.** Principles similar to those under A, may often be applied in subtraction as illustrated below:—

1. $3786437 - 579993 = 3786437 - 580000 + 7.$

2. $864379 - 298994 = 864379 - 300000 + 1006.$

3. 18 lb. 6 oz. 14 dwt. 7 gr. — 12 lb. 11 oz. 19 dwt. 23 gr. = 18 lb. 6 oz. 14 dwt. 7 gr. — 13 lb. + 1 gr.

4. 28 yd. 1 qr. 3 na. 1 in. — 21 yd. 3 qr. 3 na. 1 in. = 28 yd. 1 qr. 3 na. 1 in. — 22 yd. + 1¼ in.

5. How many are $83764 - 5998$?

6. How many are $976437 - 498991$?

7. What is the value of 17 T. 13 cwt. 5 lb. 4 dr. — 3 T. 19 cwt. 3 qr. 27 lb. 15 oz. 11 dr.?

8. What is the value of 9 w. 3 da. 15 h. 23 m. 16 sec. — 4 w. 6 da. 23 h. 59 m. 47 sec.?

**C.** To multiply by 99, observe that since $99 = 100 - 1$, 99 times a number must equal 100 times the number minus once the number. For example,

99 times 837 = 100 times 837 — 837 = $83700 - 837 = 82{,}863$.

To multiply by 999, observe that since $999 = 1000 - 1$, 999 times a number must equal 1000 times the number minus once the number. For example,

999 times 14.67 = 1000 times 14.67, — 14.67 = $14670 - 14.67 = 14655.33$.

By observing how in examples like the above the figures of the minuend and subtrahend must compare with each other, the results can be written at once.

1. Multiply 18 by 99; 25 by 99.
2. Multiply 8105 by 99; 71.15 by 99.
3. Multiply 75.074 by 99; 706 by 99.
4. Multiply 31748 by 9; 31748 by 99.
5. Multiply 31748 by 999; 31748 by 9999.
6. How many are 98 times 3587? (*Observe that* $98 = 100 - 2$.)

7. How many are 994 times 7387? (*Observe that* $994 = 1000 - 6$.)

8. How many are 997 times 7943?

9. How many are 9992 times 543876?

10. How many are 99993 times 6854297?

**D.** When the multiplier is a convenient fractional part of 10, 100, 1000, or a unit of any higher denomination, the following principles may be advantageously applied.

1. How many are 25 times 679?

*Solution.* Since 25 equals one fourth of 100, 25 times 679 must equal one fourth of 100 times 679, or, $\frac{1}{4}$ of 67900, which equals 16975.

2. How many are 25 times 657?

3. How many are 50 times 657?

4. How many are $12\frac{1}{2}$ times 834? (*Observe that* $12\frac{1}{2} = \frac{1}{8}$ *of* 100.)

5. How many are $16\frac{2}{3}$ times 957? (*Observe that* $16\frac{2}{3} = \frac{1}{6}$ *of* 100.)

6. How many are $33\frac{1}{3}$ times 871? (*Observe that* $33\frac{1}{3} = \frac{1}{3}$ *of* 100.)

7. How many are $14\frac{2}{7}$ times 249? (*Observe that* $14\frac{2}{7} = \frac{1}{7}$ *of* 100.)

8. How many are $3\frac{1}{3}$ times 249? (*Observe that* $3\frac{1}{3} = \frac{1}{3}$ *of* 10.)

9. How many are $2\frac{1}{2}$ times 822? (*Observe that* $2\frac{1}{2} = \frac{1}{4}$ *of* 10.)

10. How many are $6\frac{1}{4}$ times 944? (*Observe that* $6\frac{1}{4} = \frac{1}{16}$ *of* 100.)

11. How many are $333\frac{1}{3}$ times 9478? (*Observe that* $333\frac{1}{3} = \frac{1}{3}$ *of* 1000.)

12. How many are 125 times 8767? (*Observe that* $125 = \frac{1}{8}$ *of* 1000.)

13. How many are 250 times 6894? (*Observe that* $250 = \frac{1}{4}$ *of* 1000.)

**E.** It frequently happens that one part of the multiplier is a factor of another part. When this is the case, labor can often be saved by applying the principles illustrated in the following examples: —

1. What is the product of 7678 multiplied by 427?

By examining the multiplier we perceive that its tens equal 60 times its units, and, therefore, that the desired product can be obtained by adding to the product of the multiplicand multiplied by the units' figure of the multiplier, 60 times this product.

The following method of writing the numbers used will exhibit the process:—

7,678.
427.
53,746. = 7 times 7,678.
60 times first product = 3,224,76 . = 420 times 7,678.
3,278,506. = 427 times 7,678.

2. How many are 72144 times 874369?

By examining the multiplier, we observe that the number expressed by the figures in the three places nearest to the point, is .002 of the number expressed by the other figures. We may, therefore, perform the work by the method indicated below:—

874,369.
72,144.
62,954,568,000. = 72000 times 874,369.
.02 of first product = 125,909,136. = 144 times 874,369.
63,080,477,136. = 72144 times 874,369.

3. How many are 1875625125 times 97643721785?

97,643,721,785.
1,875,625,125.
12,205,465,223,125. = 125 times Multiplicand.
5000 times first prod. = 61,027,326,115,625, . = 625000 times Multiplicand.
3000 times second prod. = 183,081,978,346,875, , . = 1875000000 times Multiplicand.
183,143,017,878,455,848,125. = 1875625125 times Multiplicand.

4. What is the product of 76437 multiplied by 14874?
5. What is the product of 879438 multiplied by 19899?
6. What is the product of 43678 multiplied by 17525?
7. What is the product of 476437 multiplied by 83415?
8. What is the product of 9763278 multiplied by 43821973?
9. What is the product of 5428973 multiplied by 36324?
10. What is the product of 463829 multiplied by 1998999?

**F.** 1. How many times is 99 contained in 437?

*Solution.* 100 is contained 4 times in 437, with a remainder of 37; but as 99 is contained in 100 once, with a remainder of 1, it must be contained 4 times in 400, with a remainder of 4, which remainder added to 37, the remainder by the first division, gives 41 as the remainder of the division by 99. The quotient must therefore be 4, and the remainder 41.

*2d Solution.* 437 = 400 + 37; and since 100 = 99 + 1, 400 must equal 4 times 99 + 4. Adding 4 to 37, we have 41 as the remainder of the division by 99. Therefore, 437 = 4 times 99, with a remainder of 41.

2. How many times is 99 contained in 15378?

We separate the hundreds from the tens and units of the dividend by a vertical line, thus:—

$$\begin{array}{r|l} 153 & 78. \\ 2 & 31. \\ \hline 155 & 33. \end{array}$$

Then, since 100 = 99 + 1, 15300 must equal 153 times 99, + 153. Adding 153 to 78, gives 231 as the remainder of the first division by 99. This remainder being greater than 99, we divide it by 99, as we did the original dividend. 200 = 2 times 99 plus 2, and adding the 2 to 31, we have 33 as the remainder of the second division by 99; this being less than 99, the division is completed. Now, by adding 153 and 2 together, we obtain 155 as the quotient of the division by 99, and we have a remainder of 33.

In a similar manner we can divide by 999, 9999, &c.

Let the pupil find, if he can, the application of a similar principle to the division by 98, 97, 96, &c.; also to 998, 997, &c., and afterward perform the following examples:—

3. 186738 = how many times 99?
4. 49763842 = how many times 999?
5. 763852748 = how many times 9999?
6. 9842987483 = how many times 99999?
7. 54783 = how many times 98?
8. 2987637 = how many times 96?
9. 248763 = how many times 997?
10. 69874325 = how many times 9998?

**G.** 1. 7987 = how many times 25?

*Solution.* 7987 = 7900 + 87. Since in 100, 25 is contained 4 times, in 7900 it must be contained 79 times 4 times, or 316 times; and since in 87 it is contained 3 times, with a remainder of 12, in 7987 it must be contained 316 + 3, or 319 times, with a remainder of 12. Therefore, 7987 = $319\frac{12}{25}$ times 25.

2. 86794 = how many times 25?
3. 3475 = how many times $12\frac{1}{2}$?
4. 6950 = how many times $16\frac{2}{3}$?
5. 42725 = how many times $6\frac{1}{4}$?
6. 54737 = how many times 125?
7. 98734 = how many times $33\frac{1}{3}$?
8. 4765465 = how many times $333\frac{1}{3}$?

**H.** Two numbers whose tens are alike, may be multiplied together on the principles involved in the following explanation of the multiplication of 38 by 34: —

34 times 38 = 34 times 30 + 34 times 8; but as 34 times 8 = 8 times 34 = 8 times 30 + 8 times 4, 34 times 38 = 34 times 30 + 8 times 30 + 8 times 4 = 42 times 30 + 8 times 4; or it equals *40 times 30 + 2 times 30 + 8 times 4 = 1200 + 60 + 32 = 1292.

Now, as the same principles will apply to the multiplication of any two numbers similar to the above, we infer that to find the product of two numbers whose tens are alike, we may add the units of one factor to the other factor, multiply the sum by the tens of either factor, and to this product add the product of the units.

*Applications illustrated.*

1. $49 \times 44 = 53 \times 40 + 9 \times 4 = 50 \times 40 + 3 \times 40 + 9 \times 4$.
2. $83 \times 83 = 83^2 = 86 \times 80 + 3 \times 3 = 80 \times 80 + 6 \times 80 + 3 \times 3$.
3. $67 \times 63 = 70 \times 60 + 7 \times 3$.
4. $98 \times 92 = 100 \times 90 + 8 \times 2$.
5. $35^2 = 40 \times 30 + 5 \times 5$.

---

* This form is usually regarded as more convenient than the one preceding. In practice, results only should be named. Thus, in multiplying 38 by 34, we may say 1200, 1260, 1292. A little practice in this way will enable the pupil to give the final result at once. He should write no figures in performing the work.

6. $78^2 = 86 \times 70 + 8 \times 8 = 80 \times 70 + 6 \times 70 + 8 \times 8$.

Apply these principles to the following examples: —

7. $87 \times 83$? $58 \times 53$? $67 \times 63$? $84 \times 89$?
8. $75 \times 75$? $29 \times 28$? $38 \times 37$? $82 \times 88$?
9. $44 \times 46$? $54 \times 56$? $72 \times 78$? $93 \times 97$?
10. $89 \times 81$? $86 \times 85$? $96 \times 98$? $49 \times 47$?
11. $24^2$? $46^2$? $87^2$? $65^2$? $32^2$?
12. $93^2$? $63^2$? $46^2$? $21^2$? $93^2$?
13. $57^2$? $74^2$? $35^2$? $85^2$? $64^2$?
14. $86^2$? $96^2$? $76^2$? $38^2$? $48^2$?
15. $29^2$? $89^2$? $69^2$? $22^2$? $62^2$?

**I.** When the units' figure is the same in both factors, we may apply the principles illustrated in the following multiplication of 43 by 83: —

43 times 83 = 40 times 80 + 40 times 3 + 3 times 80, or 80 times 3 + 3 times 3 = 40 times 80 + 120 times 3, or 3 times 120 + 3 times 3 = 3200 + 360 + 9 = 3569.

As the same principles will apply to the multiplication of any numbers similar to the above, we infer that the product of any two numbers having the same units' figure, must equal the product of the tens by the tens, plus the product of the sum of the tens by the units, plus the square of the units.

Show the truth of the following equations: —

67 times 37 = 60 times 30 + 7 times 90 + 7 times 7.
85 times 45 = 80 times 40 + 5 times 120 + 5 times 5.
47 times 67 = 40 times 60 + 7 times 100 + 7 times 7.
86 times 26 = 80 times 20 + 6 times 100 + 6 times 6.
84 times 84 = 80 times 80 + 4 times 160 + 4 times 4.

Apply these principles to the following examples: —

$87 \times 27$? $53 \times 83$? $69 \times 49$? $74 \times 34$?
$68 \times 28$? $81 \times 71$? $28 \times 98$? $64 \times 84$?
$29 \times 89$? $73 \times 63$? $85 \times 95$? $28 \times 68$?
$27 \times 67$? $84 \times 24$? $92 \times 12$? $67 \times 47$?

**J.** The following illustrates a method that can often be advantageously applied, when one factor is the sum of two numbers of which the other is the difference.

1. How many are 28 times 32 ?

Since $28 = 30 - 2$, and $32 = 30 + 2$, it follows that 28 times $32 = 30$ times $32 - 2$ times $32 = 30$ times $30 + 30$ times $2 - 2$ times $30 - 2$ times 2. But it is evident that if we add one number to another, and from the sum subtract the number which we added, we shall have, as a result, the number to which the addition was made. In the above example, we have to add 2 times 30, and to subtract its equivalent 30 times 2; but, as the result will not be affected by these operations, they may be omitted. Therefore, we have 28 times $32 = 30$ times $30 - 2$ times $2 = 30^2 - 2^2 = 900 - 4 = 896$.

As these principles will apply to the multiplication of all numbers similar to the above, we infer that the product of two factors, one of which is the sum of two numbers, and the other their difference, is equal to the difference of their squares.

Show the truth of the following equations : —

$87 \times 93 = 90^2 - 3^2$.
$42 \times 58 = 50^2 - 8^2$.
$109 \times 91 = 100^2 - 9^2$.
$412 \times 388 = 100^2 - 12^2$.

Apply these principles to the following examples : —

$64 \times 76$ ? $83 \times 97$ ? $41 \times 59$ ? $63 \times 57$ ?
$87 \times 93$ ? $204 \times 196$ ? $25 \times 35$ ? $607 \times 593$ ?
$785 \times 815$ ? $34 \times 26$ ? $42 \times 38$ ? $91 \times 89$ ?
$55 \times 65$ ? $425 \times 375$ ? $809 \times 791$ ? $693 \times 707$ ?
$27 \times 33$ ? $51 \times 69$ ? $512 \times 488$ ? $904 \times 896$ ?

**K.** We give the following rules for squaring numbers ending in 25 and 75, leaving the pupil to exercise his ingenuity in finding their demonstration.

To square a number ending in 25, multiply the number representing the hundreds by the same number increased by $\frac{1}{2}$, call the result ten-thousands, and add to it 625.

1. $825^2 = 8\frac{1}{2}$ times 8 ten-thousands $+ 625 = 680000 + 625 = 680625$.

2. $725^2 = 7\frac{1}{2}$ times 7 ten-thousands $+ 625 = 525000 + 625 = 525625$.

3. What is the square of 225 ? of 925 ? of 325 ? of 525 ? of 425 ? of 125 ? of 1125 ? of 1225 ? of 1425 ? of 625 ? of 1325 ?

To square a number ending in 75, multiply the number representing the hundreds by the same number increased by $1\frac{1}{2}$, call the result ten-thousands, and add to it 5625.

4. $775^2 = 8\frac{1}{2}$ times 7 ten-thousands $+ 5625 = 595000 + 5625 = 600625$.

5. $475^2 = 5\frac{1}{2}$ times 4 ten-thousands $+ 5625 = 220000 + 5625 = 225625$.

6. What is the square of 975? of 275? of 875? of 1275? of 175? of 575? of 375? of 1175? of 675? of 1075? of 1375?

**L.** The contractions before mentioned may frequently be applied in the method illustrated in the following equations and examples: —

1. 126 times a number = 125 times the number + the number.
2. 124 times a number = 125 times the number — the number.
3. $33\frac{2}{3}$ times a number $= 33\frac{1}{3}$ times the number $+ \frac{1}{3}$ of the number.
4. $87\frac{1}{2}$ times a number = 100 times the number — $12\frac{1}{2}$ times the number.
5. 64 times 57 = 64 times 56 + 64, or it equals 63 times 57 + 57.
6. 47 times 52 = 47 times 53 — 47, or it equals 48 times 52 — 52.
7. 78 times 84 = 78 times 82 + 78 times 2, or it equals 76 times 84 + 2 times 84.
8. How many are 124 times 6737?
9. How many are 126 times 54797?
10. How many are $33\frac{2}{3}$ times 97848?
11. How many are $87\frac{1}{2}$ times 64734?
12. How many are $166\frac{1}{3}$ times 8352?
13. How many are $93\frac{3}{4}$ times 6832?
14. How many are 17625 times 987439? *Suggestion.* $176 = 7 \times 25 + 1$.
15. How many are 37253 times 784362? *Suggestion.* $372 = 7 \times 53 + 1$.
16. How many are 64772 times 5943786? *Suggestion.* $647 = 9 \times 72 - 1$.

17. How many are 93 times 86? 44 times 37?
18. How many are 24 times 38? 42 times 36?
19. How many are 96 times 83? 87 times 76?
20. How many are 73 times 68? 52 times 44?

**M.** When several factors are to be multiplied together, care should be taken to arrange them in such order as to render the work as simple as possible. A habit of examining each question carefully before attempting its numerical solution is one of the most valuable a pupil can acquire. A distinguished teacher once said, "If I were required, on peril of my life, to perform a complicated problem in two minutes, I would spend the first minute in considering how to do it." The following equations and examples furnish illustrations of principles that can sometimes be applied: —

1. $4 \times 7 \times 5 \times 3 \times 5 \times 29 = (4 \times 5 \times 5) \times (3 \times 7) \times 29 = 100 \times 21 \times 29 = 60900$.

*Explanation.* In this example, as we have among the factors 4, 5, and 5, the product of which is 100, it is most convenient to consider those factors by themselves. We then have for the remaining factors, 7, 3, and 29, the product of which equals 21 times 29 = 609, which, multiplied by 100, = 60900.

2. $16\frac{2}{3} \times 7 \times 16 \times 5 \times 3 \times 12\frac{1}{2} = (16\frac{2}{3} \times 3 \times 2) \times (12\frac{1}{2} \times 8) \times 7 \times 5 = 100 \times 100 \times 35 = 350000$.

*Explanation.* In solving this example, we observe that $16\frac{2}{3} = \frac{1}{6}$ of 100, and that $12\frac{1}{2} = \frac{1}{8}$ of 100. By dividing 16 into the factors 8 and 2, and considering the factors $16\frac{2}{3}$, 3, and 2 together, and the factors 8 and $12\frac{1}{2}$ together, we have the product equal to $100 \times 100 \times 35 = 350000$.

NOTE. The contractions in such examples as the two immediately preceding this note, are made by so arranging the factors as to get for a product a number by which we can easily multiply; as, for instance, 100, 1000, 1001, &c. $1001 = 7 \times 11 \times 13$, a fact which the pupil should remember, and apply in every instance that admits of it. Persons unaccustomed to look for contractions would be surprised to see how many there are, and how frequently they can be applied to advantage. Scarcely an example presents itself, that does not admit of some contraction or other. Practice will make them familiar and easy to all.

3. $11 \times 23 \times 7 \times 6 \times 13 = (11 \times 7 \times 13) \times (6 \times 23) = 1001 \times 138 = 138138$.

4. $39 \times 55 \times 28 \times 235 = 3 \times 13 \times 5 \times 11 \times 7 \times 4$

$\times 5 \times 47 = (13 \times 11 \times 7) \times (4 \times 5 \times 5) \times (3 \times 47) = 1001 \times 100 \times 141 = 14114100$.

*Explanation.* Observing that $39 = 3 \times 13$, that $55 = 5 \times 11$, that $28 = 4 \times 7$, we select from these the factors 13, 11, and 7, the product of which is 1001. From the remaining factors, since $235 = 5 \times 47$, we may select the factors 5, 5, and 4, the product of which is 100. The remaining factors being 3 and 47, the product of which is 141, we have for the complete product $141 \times 100 \times 1001 = 14114100$.

NOTE. The pupil should perform and explain most, if not all, of the questions in this and the next letter, without writing any figures.

5. What is the product of $24 \times 47 \times 33\frac{1}{3} \times 125$?
6. What is the product of $625 \times 947 \times 16$?
7. What is the product of $91 \times 9 \times 132 \times 8$?
8. What is the product of $86 \times 166\frac{2}{3} \times 154 \times 78$?
9. What is the product of $37\frac{1}{2} \times 15 \times 64 \times 275$?
10. What is the product of $63 \times 37\frac{1}{2} \times 66\frac{2}{3} \times 44\frac{4}{9}$?
11. What is the product of $55 \times 1376 \times 65 \times 35$?
12. What is the product of $87\frac{1}{2} \times 104 \times 4796$?

**N.** The principles illustrated below can frequently be applied to advantage.

1. $6 \times 84 + 7 \times 43 + 47 \times 84 + 17 \times 84 + 18 \times 43 = (6 + 47 + 17) \times 84 + (7 + 18) \times 43 = 70 \times 84 + 25 \times 43 = 5880 + 1075 = 6955$.

2. $47 \times 28 + 36 \times 62 + 28 \times 23 - 29 \times 62 - 66 \times 28 - 62 \times 2 = (47 + 23 - 66)\, 28 + (36 - 29 - 2) \times 62 = 4 \times 28 + 5 \times 62 = 112 + 310 = 422$.

3. What is the value of $7 \times 64 + 9 \times 75 - 3 \times 64 - 5 \times 75 + 8 \times 64 + 4 \times 75$?

4. What is the value of $9 \times 93 + 27 \times 6 + 88 \times 93 + 17 \times 27$?

5. What is the value of $84 \times 17 + 36 \times 4 + 36 \times 84 + 12 \times 36 - 3 \times 84$?

6. What is the value of $9 \times 13 + 16 \times 9 + 8 \times 31 + 9 \times 23 + 23 \times 8 + 21 \times 7 + 43 \times 9$?

7. What is the value of $25 \times 19 + 12\frac{1}{2} \times 7 + 25 \times 21 + 29 \times 12\frac{1}{2}$?

8. What is the value of $8 \times 16\frac{2}{3} + 17 \times 33\frac{1}{3} + 14 \times 6\frac{1}{4} + 9 \times 12\frac{1}{2}$?

*Suggestions.* $8 \times 16\frac{2}{3} = 4 \times 2 \times 16\frac{2}{3} = 4 \times 33\frac{1}{3}$. $14 \times 6\frac{1}{4} = 7 \times 2 \times 6\frac{1}{4} = 7 \times 12\frac{1}{2}$. Therefore, the value of the above expression $= (4 + 17) \times 33\frac{1}{3} + (7 + 9) \times 12\frac{1}{2} = 700 + 200 = 900$.

9. What is the value of $18 \times 12\frac{1}{2} + 7 \times 25 + 28 \times 75$?

*Suggestions.* $18 \times 12\frac{1}{2} = 9 \times 25$. $28 \times 75 = 28 \times 3 \times 25 = 84 \times 25$.

10. What is the value of $46 \times 12 + 19 \times 24 + 27 \times 48 + 16 \times 96 + 19 \times 72 + 83 \times 24$?

11. I bought 17 lb. of raisins at \$.09 per lb., 28 lb. of flour at \$.04 per lb., 4 lb. of butter at \$.23 per lb., 37 lb. of cheese at \$.09 per lb., and 9 lb. of tea at \$.43 per lb. What was the amount of my purchase?

12. Mr. Elijah Wheelock, Jr., bought of Mr. Elbridge Clapp, 7 bags of corn at \$1.50 per bag, 9 bushels of oats at \$.37½ per bu., 12½ lb. of rice at \$.05 per lb., 150 lb. of sugar at \$.09 per lb., and 25 yd. of sheeting at \$.08 per yd. What was the value of the goods he purchased?

13. Miss Lucy Smith bought of Jones & Brown, 9 yd. of black silk at \$.87½ per yd., 6 yd. of muslin at \$.42 per yd., 8 skeins of sewing silk at \$.06 per skein, and 3 pair of kid shoes at \$.87½ per pair. What was the amount of her purchase?

**O.** 1. How much will 13 cwt. 2 qr. 7 lb. of sugar cost at \$8.64 per cwt.?

*Direction.* First get the cost of 13 cwt.; then of 2 qr. or ½ of a cwt.; and then of 7 lb. or ⅛ of 2 qr. The work may be written as below: —

| | | |
|---|---|---|
| | \$8.64 = | cost of 1 cwt. |
| 13 times \$8.64 = | \$112.32 = | " " 13 cwt. |
| ½ of \$8.64 = | 4.32 = | " " 2 qr. |
| ⅛ of \$4.32 = | .54 = | " " 7 lb. |
| | \$117.18 = | " " 13 cwt. 2 qr. 7 lb. |

2. How much will 26 A. 1 R. 25 sq. rd. of land cost, at $174.25 per acre?

$174.25 = cost of 1 A.
26 times $174.25 = $4530.50 = " " 26 A.
$\frac{1}{4}$ of $174.25 = 43.562 = " " 1 R.
$\frac{1}{2}$ of $43.562 = 21.781 = " " 20 sq. rd.
$\frac{1}{4}$ of $21.781 = 5.445 = " " 5 sq. rd.
$4601.288 = " " 26 A. 1 R. 25 sq. rd.

NOTE. In dividing, in examples like the above, it is unnecessary to carry out the work further than to mills.

3. How much will 16 cwt. 3 qr. 21 lb. of tea cost, at $47.83 per cwt.?

*Suggestion.* Since 16 cwt. 3 qr. 21 lb. lacks but 7 lb. of being 17 cwt., its cost must equal the cost of 17 cwt. minus the cost of 7 lb. We have, therefore, the following written work:—

$47.83 = cost of 1 cwt.
17 times $47.83 = $813.11 = " " 17 cwt.
$\frac{1}{16}$ of $47.83 = 2.989 = " " 7 lb. to be subtracted.
$810.121 = " " 16 cwt. 3 qr. 21 lb.

4. How much will it cost to build a road 37 m. 5 fur. 25 rd. in length, at the rate of $2173.75 per mile?

5. How much will 47 yd. 2 qr. 3 na. of broadcloth cost, at $3.125 per yd.?

6. How much will 22 hhd. 21 gal. 3 qt. 1 pt. of wine cost, at $47.875 per hhd., if each hogshead contains 63 gallons?

7. How much will 24 bu. 1 pk. 6 qt. of wheat cost, at $1.375 per bushel?

8. How much will 37 bu. 3 pk. 4 qt. of wheat cost, at $1.625 per bushel?

9. How much will 14 cwt. 2 qr. 15 lb. of coffee cost, at $12.625 per cwt., the cwt. being reckoned at 100, and the qr. at 25 lb.?

10. A man bought a farm, containing 137 A. 3 R. 30 sq. rd., at $46.94 per acre. How much did it cost him?

11. A trader bought 184 bu. 2 pk. 6 qt. of Indian corn, at $.875 per bushel. How much did it cost him?

12. An English merchant purchased 1728 yd. of cloth, at 1s. 8d. per yd. What was the cost of the whole?

*Suggestion.* 1s. 8d. = $\frac{1}{12}$ of 1£.

13. What will 857 yd. of broadcloth cost, at 16s. 8d. per yd.?

*Suggestions.* 16s. 8d. = 1£ — 3s. 4d. = 1£ — $\frac{1}{6}$£.

Again: 16s. 8d. = 10s. + 6s. 8d. = $\frac{1}{2}$£ + $\frac{1}{3}$£.

Adopting the first of the above suggestions, the work would be written thus: —

857£ = cost at 1£ per yd.
$\frac{1}{6}$ of 857£ = 142£, 16s. 8d. = cost at 3s. 4d. per yd.
By subtraction, 714£, 3s. 4d. = cost at 16s. 8d. per yd.

Adopting the second of the above suggestions, the work would be written thus: —

857£ = cost at 1£ per yd.

$\frac{1}{2}$ of 857£ = 428£, 10s. = cost at 10s. per yd.
$\frac{1}{3}$ of 857£ = 285£, 13s. 4d. = cost at 6s. 8d. per yd.
714£, 3s. 4d. = cost at 16s. 8d. per yd.

14. What will 1478 reams of paper cost, at 13s. 4d. per ream?

*Suggestion.* 13s. 4d. = 1£ — 6s. 8d. = 1£ — $\frac{1}{3}$£.

15. What will 737 yd. of black silk cost, at 17s. 6d. per yd.?

*Suggestion.* 2s. 6d. = $\frac{1}{8}$ of 1£.

16. If 14 yd. 1 qr. 2 na. 1 in. of cloth cost 12£, 13s. 8d., what will 28 yd. 3 qr. 2 in. cost?

*Suggestion.* 28 yd. 3 qr. 2 in. = twice 14 yd. 1 qr. 2 na. 1 in.

17. If 7 lb. 3 oz. 4 dr. of sugar cost 2s. 9d. 1 qr., what will 21 lb. 9 oz. 12 dr. cost?

18. If 17 A. 3 R. 7 rd. of land cost 19£, 14s. 9d., what will 88 A. 3 R. 35 rd. cost?

19. If 2 lb. 10 oz. 18 dwt. 22 gr. of gold can be bought for $667.75, for how much can 1 lb. 5 oz. 9 dwt. 11 gr. be bought?

20. If 23 lb. 13 oz. 8 dr. of potash can be bought for $2.12, how much can be bought for $16.96?

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## RECOMMENDATIONS.

*From J. W. Upton, Esq., now Teacher of English Department of Pinkerton Academy, Derry, N. H.*

Greenfield, February 26, 1850.

Gentlemen: During the present term of my school, my assistant has used the copy of your Arithmetic which you had the kindness to present me. I find that the scholars (though not provided with books) have attained a greater facility in the performance of examples in the different text books used by them in their several classes; that no other arithmetic takes so well with them; that more interest is manifested in recitation; and that the mental discipline is not only good, but practical.

Satisfied that much has been done with one book, and that if our scholars can each have a book of the same kind more will be done, I shall earnestly endeavor to have our committee adopt your Arithmetic as a text book.

Yours respectfully, JOSEPH W. UPTON.

*From N. Tillinghast, Esq., Principal of Bridgewater Normal School.*

Bridgewater, May 11, 1852.

D. P. Colburn, Esq. Dear Sir: I have used your "Decimal Arithmetic" with a class of my pupils from the commencement of the Written Arithmetic to the end of the volume, and therefore feel able to speak of it with some degree of confidence.

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